11 Übung

Mathe 1

19.12.2023

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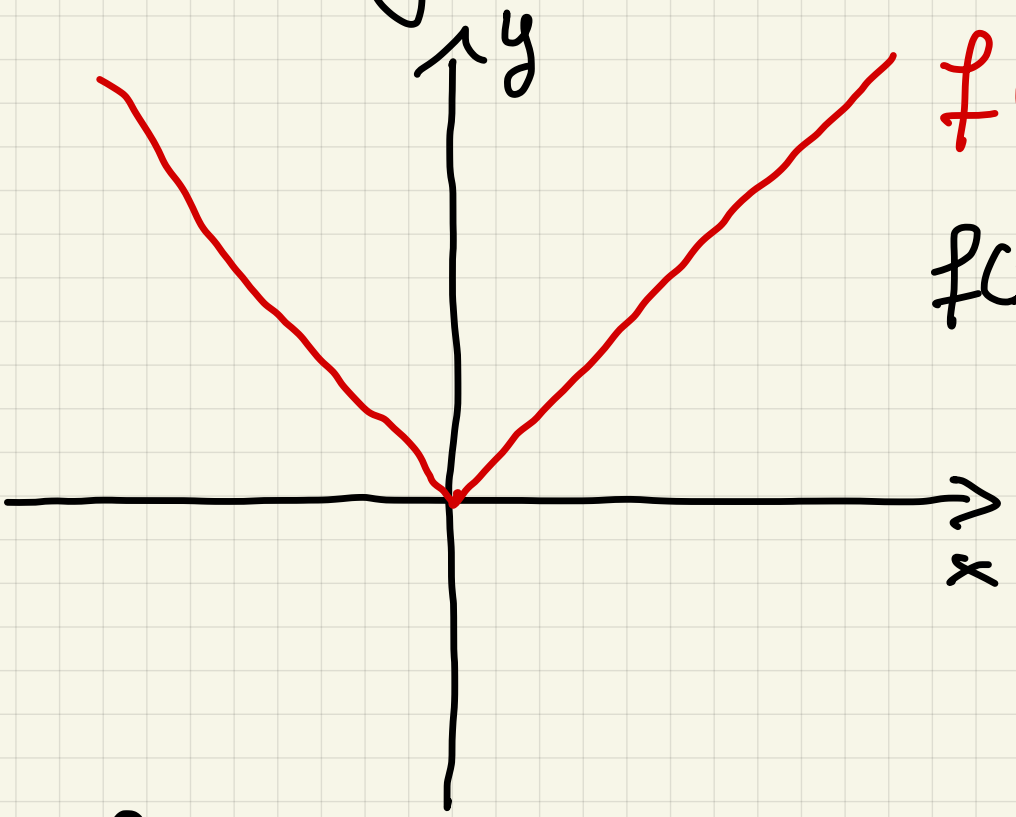


$f(x) = |x|$

$$f(x) = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$f(x) = |x|$ stetig

Kritisches Punkt $x = 0$

$$\lim_{x \to 0^+} = 0$$

$$\lim_{x \to 0^-} = 0$$

$$f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} f'(x) = 1$$

$$\lim_{x \to 0^-} f'(x) = -1$$

Aufgabe 4.2 b

$$f(x) = \sin(x) \cdot \cos(x)$$

$$= \frac{1}{2} \sin(2x) \cdot$$

Additionstheorem

$$f'(x) = \frac{1}{2} \cdot 2 \cdot \cos(2x)$$

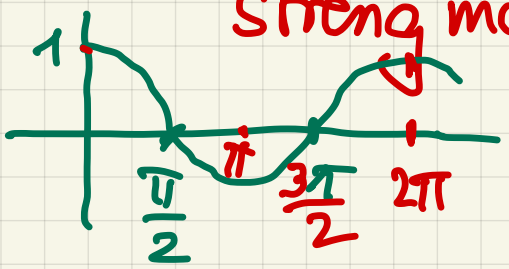
$$= \cos(2x)$$

$$f'(x) = \begin{cases} > 0 & \text{für } -\frac{\pi}{4} + k\pi < x \\ \leq 0 & \text{"fallend"} \quad < \frac{\pi}{4} + k\pi \end{cases}$$

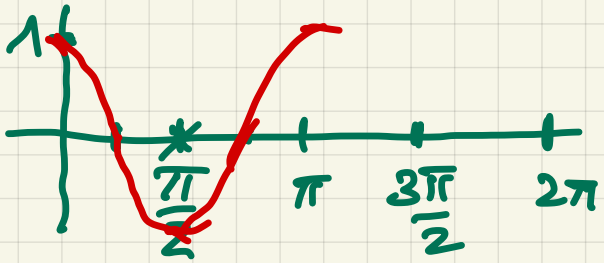
$k \in \mathbb{Z}$

streng monoton steigend

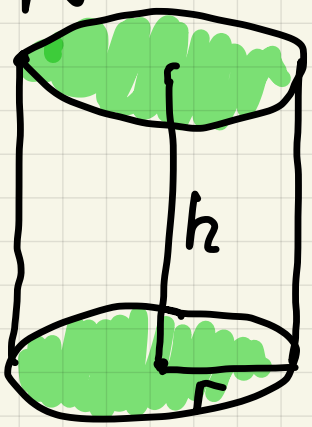
$f(x) = \cos x$



$f(x) = \cos(2x)$



Aufgabe 4.6 Extremwertaufgabe



$V = \pi \cdot r^2 \cdot h$

Nebenbedingung

$O(r;h) = 2\pi \cdot r^2 + 2\pi r \cdot h$

Zielfunktion — Boden + Deckel — Mantel

Aus $V = \pi \cdot r^2 \cdot h \Rightarrow h = \frac{V}{\pi \cdot r^2}$ *

mit *

$O(r) = 2\pi \cdot r^2 + 2\pi \cdot r \cdot \frac{V}{\pi \cdot r^2}$

$\rightarrow \frac{2V}{r}$

Minimum für $O(r)$

$O'(r) = 4\pi \cdot r - 2V \cdot r^{-2}$

NR

$\frac{2V}{r} = 2V \cdot r^{-1}$

$\left(\frac{2V}{r}\right)' = (2V \cdot r^{-1})'$

$= -1 \cdot 2Vr^{-2}$

$O'(r) = 0$

$4\pi \cdot r - 2V \cdot r^{-2} = 0$

$O''(r) = 4\pi + 4V \cdot r^{-3}$

$> 0 \Rightarrow$ MIN

Nebenbemerkung

$$f(x) = 5x^3 + 3abx^2 + \sin(x)$$

$$f'(x) = 3 \cdot 5 \cdot x^{3-1} + 2 \cdot 3abx^{2-1} + \cos(x)$$

$$= 15x^2 + 6abx + \cos(x)$$

$$\left(\frac{2v}{r}\right)'$$

$u = 2v$

$v = r$

$u' = 0$

$v' = 1$

$$\frac{u' \cdot v - u \cdot v'}{r^2 \; v^2}$$

$$\frac{0 \cdot r - 2v \cdot 1}{r^2 \; v^2}$$

Null

$$= \frac{0 - 2v}{r^2} = -\frac{2v}{r^2} = -2v \cdot r^{-2}$$

$$r = \sqrt[3]{\frac{V}{2\pi}} \quad *$$

$$\frac{h}{r} = \frac{\frac{V}{\pi \cdot r^2}}{r} = \frac{V}{\pi \cdot r^3}$$

$$\overset{=}{*} \frac{V}{\pi \cdot \frac{V}{2\pi}} = \frac{V}{\frac{V}{2}} = 2$$

stetig: "ohne Unterbrechung zu zeichnen"

differenzierbar: "ohne Ecken und Kanten zu zeichnen"