Faktorenzerlegung

$a \cdot b = 0 \Leftrightarrow a = 0 \lor b = 0$

Bsp. $5a - 25a^2 = 0$

$\Leftrightarrow 5a(1 - 5a) = 0$

$\Leftrightarrow 5a = 0 \lor 1 - 5a = 0$

$\Leftrightarrow a = 0 \lor a = \frac{1}{5}$

Ü a) $x^2 - 25 = 0$

$\Leftrightarrow (x-5)(x+5) = 0$

$\Leftrightarrow x = 5 \lor x = -5$

b) $x \cdot \ln(x^2+1) + x^2 \cdot \ln(x^2+1) = 0$

$\Leftrightarrow x \cdot \ln(x^2+1)(1 + x) = 0$

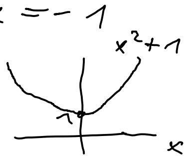
S2-9

$\Leftrightarrow x = 0 \lor \ln(x^2+1) \lor 1 + x = 0$

$\Leftrightarrow x = 0 \lor x^2+1 = 1 \lor x = -1$

$\Leftrightarrow x = 0 \lor x = 0 \lor x = -1$

$D = \mathbb{R}$ $\qquad L = \{0, -1\}$



c) $x \ln(x) + x^2 \ln(x^2) = 0$

$\Leftrightarrow x \ln(x) + x^2 \cdot 2 \ln(x) = 0$

$\Leftrightarrow x \ln(x)(1 + 2x) = 0$

$\Leftrightarrow x = 0 \lor \ln(x) = 0 \lor 1 + 2x = 0$

$\Leftrightarrow \cancel{x = 0} \lor x = 1 \lor \cancel{x = -\frac{1}{2}}$

$D = \mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$

$\Rightarrow L = \{1\}$

weil $\ln(x)$ nur für positive x definiert

Summen

$\sum_{k=1}^{4} 5 = 5+5+5+5 = 4 \cdot 5 = 20$

$\sum_{k=-2}^{2} 5 = 5+5+5+5+5 = 25$

$\left.\begin{matrix} o = 2 \\ u = -2 \end{matrix}\right\} S = 2 - (-2) + 1 = 5$

Ü a) $\sum_{k=1}^{50} k^4 - \sum_{k=4}^{54} (k-2)^4$

$= 1^4 + \cancel{2^4} + \cancel{\ldots} + \cancel{49^4} + \cancel{50^4}$

$-(\cancel{2^4} + \cancel{3^4} + \cancel{\ldots} + \cancel{50^4} + 51^4 + 52^4)$

$= 1^4 - 51^4 - 52^4 \approx -14 \cdot 10^6$

b) $\sum_{i=1}^{50} i(i-1) - \sum_{i=3}^{52} i(i+1)$

$= 1 \cdot 0 + 2 \cdot 1 + \cancel{\ldots} + \cancel{50 \cdot 49}$

$-(\cancel{3 \cdot 4} + \ldots + \cancel{49 \cdot 50} + 52 \cdot 53 + 51 \cdot 52 + 50 \cdot 51)$

$= 1 \cdot 0 + 2 \cdot 1 + 3 \cdot 2 - 50 \cdot 51 - 51 \cdot 52 - 52 \cdot 53$

$= -7950$

Ü Doppelsummen

a) $\sum_{k=1}^{2}\sum_{i=1}^{10} k\cdot i = \sum_{k=1}^{2}\left(\sum_{i=1}^{10} ki\right)$

$= \left(\sum_{k=1}^{2} k\right)\cdot\left(\sum_{i=1}^{10} i\right)$

$= 3 \cdot (1+ \dots + 10)$

$= 3 \cdot 55 = \underline{\underline{165}}$

b) $\sum_{k=1}^{3}\sum_{i=1}^{4}\left((k\cdot i)^2+5\right)$

$\overset{S2-11}{=} \left(\sum_{k=1}^{3}\sum_{i=1}^{4} k^2\cdot i^2\right) + \left(\sum_{k=1}^{3}\sum_{i=1}^{4} 5\right)$

$= \left(\sum_{k=1}^{3} k^2\right)\cdot\left(\sum_{i=1}^{4} i^2\right) + \left(\sum_{k=1}^{3} 4\cdot 5\right)$

$= (1^2+2^2+3^2)\cdot(1^2+2^2+3^2+4^2) + 3\cdot 4\cdot 5$

$= 14 \cdot 30 + 3\cdot 20$

$= \underline{\underline{480}}$

$\boxed{\begin{array}{l} \text{N.B. Bei Summen auf Klammern achten} \\ \sum_{m=1}^{5} 2m+3 \quad \text{ist ambivalent} \\ \text{entweder } \left(\sum_{m=1}^{5} 2m\right) + 3 = 33 \\ \text{oder } \sum_{m=1}^{5}(2m+3) = 45 \end{array}}$

Fakultät u. Binomialkoeff.

$$\sum_{k=1}^{3} a_k = a_1 + a_2 + a_3$$

$$\prod_{k=1}^{3} a_k = a_1 \cdot a_2 \cdot a_3$$

Fakultät $n! = 1\cdot 2\cdot \ldots \cdot (n-1)\cdot n \left(= \prod_{i=1}^{n} i\right)$ für $n>0$

und $0! = 1$

Bsp. Binomialkoeff.

$$\binom{n}{0} = \frac{\cancel{n!}}{0!\,\cancel{(n-0)!}} = \frac{1}{0!} = 1$$

$$\binom{n}{n} = \frac{\cancel{n!}}{\cancel{n!}\,(n-n)!} = \frac{1}{0!} = 1$$

$$\binom{n}{k} = \binom{n}{n-k}$$

Pascal'sches Dreieck

$$\begin{array}{lccccccc}
 & & & & k=0 & & & \\
n=0 \longrightarrow & & & & 1 & & k=1 & \\
n=1 \longrightarrow & & & 1 & & 1 & & k=2 \\
n=2 \longrightarrow & & 1 & & 2 & & 1 & \quad k=3 \\
n=3 \rightarrow & 1 & & 3 & & 3 & & 1
\end{array}$$

Additionstheorem S2-13 beweisen

$$\binom{n}{k} + \binom{n}{k+1}$$

$$= \frac{(k+1)}{(k+1)} \cdot \frac{n!}{k!\,(n-k)!} + \frac{n!}{(k+1)!\,(n-k-1)!} \cdot \frac{(n-k)}{(n-k)}$$

$$\boxed{\text{H.N.} \quad \underbrace{(k+1)\cdot k!}_{(k+1)!} \quad \underbrace{(n-k)\,(n-k-1)!}_{(n-k)!}}$$

$$= \frac{(k+1)\cdot n! + n!\,(n-k)}{(k+1)!\,(n-k)!}$$

$$= \frac{\cancel{k\,n!} + 1\cdot n! + n\cdot n! - \cancel{k\,n!}}{(k+1)!\,(n-k)!}$$

$$= \frac{(n+1)\,n!}{(k+1)!\,((n+1)-(k+1))!}$$

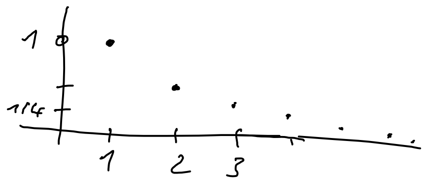
$$= \frac{(n+1)!}{(k+1)!\,((n+1)-(k+1))!} = \binom{n+1}{k+1}$$

Folgen

Bsp 1) $a_n = \frac{1}{n}$

$(a_n) = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

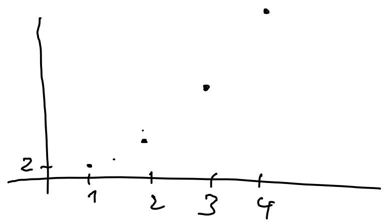
streng monoton fallend
n.o.b. , z.B. $K = 1$
n.u.b. , z.B. $k = 0$
oder $k = -1$



2) $a_n = 2^n$

$(a_n) = 2, 4, 8, 16, \dots$

streng monoton steigend
n.u.b. , z.B. $k = 0$
nicht n.o.b.

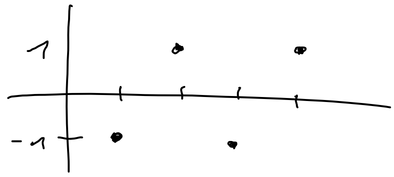


3) $a_n = (-1)^n$

$(a_n) = -1, +1, -1, +1, \dots$

alternierende Folge

nicht monoton
n.o.b. $K = 1$
n.u.b. $k = -1$



4) rekursive Folge:

$a_1 = \frac{1}{4}$, $a_{n+1} = a_n^2 + \frac{1}{4}$ $\forall n \geq 1$

$(a_n) = \frac{1}{4}, \frac{5}{16}, \frac{89}{256}, \dots$

5) Dezimalzahl

$(a_n) = 0.9,\ 0.99,\ 0.999, \dots \longrightarrow 0.\overline{9} = 1$