V MA 1 – 31.10.2018

Orga

07.11. V WK
14.11. V Arne Schnitter
21.11. Profil^2 keine V, kein Ü
28.11. V WK
05.12 V WK
ab 12.12 V Arne Schnitter

Tutorien: gering bis mäßig besucht

Mi 16^{30} Leonie Eichler 11 – 4
Fr 9–11 Jan Schröder ~10
Yannick Dittmar

Zahlenfolgen

Beispiele $(n \in \mathbb{N})$

a) $a_n = \frac{1}{n} \in \mathbb{R}$

$(a_n) = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots$

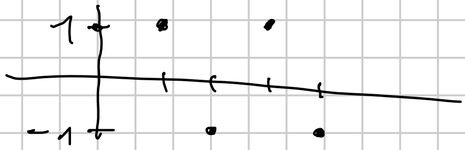
streng monoton fallend

n.o.b., z.B. $K = 1$

n.u.b., z.B. $L = 0$

b) $a_n = (-1)^n$ (alternierend)

$(a_n) = -1, +1, -1, +1, \ldots$

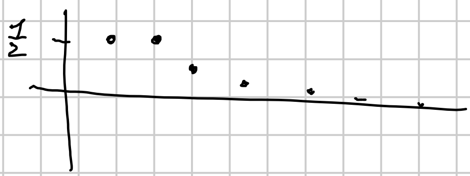


keine Monotonie

beschränkt, z.B. $K = 1$, $L = -1$

c) $a_n = \frac{n}{2^n}$

$(a_n) = \frac{1}{2}, \underset{\underset{\frac{1}{2}}{\|}}{\frac{2}{4}}, \frac{3}{8}, \frac{4}{16}$



monoton fallend

beschränkt $L = 0$, $K = 1$

d) $a_n = n$

$(a_n) = 1, 2, 3, \ldots$

streng monoton wachsend

n.u.b. $(L = 0)$

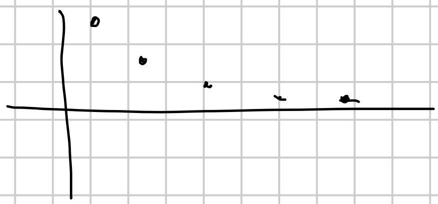
nach oben unbeschränkt

Bsp. zu Grenzwert $(n \to \infty)$

a) $a_n = \frac{1}{n}$

$\lim\limits_{n \to \infty} a_n = 0$ "Nullfolge"

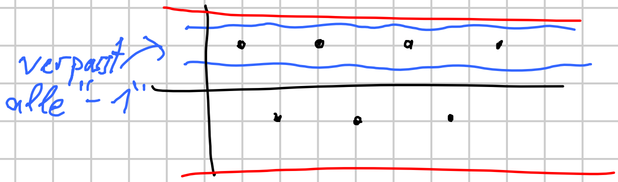
$\frac{1}{n} \xrightarrow[n \to \infty]{} 0$



b) $a_n = (-1)^n$

$\lim_{n\to\infty} a_n =$ ex. nicht

<u>divergent</u>



c) $a_n = \frac{2n-1}{3n}$

$(a_n) = \frac{1}{3}, \frac{3}{6}, \frac{5}{9},$

$$\lim_{n\to\infty} \frac{2n-1}{3n} = \lim_{n\to\infty} \frac{\cancel{n}\left(2-\underbrace{\frac{1}{n}}_{\xrightarrow{n\to\infty} 0}\right)}{\cancel{n}\cdot 3} = \frac{2-0}{3} = \underline{\underline{\frac{2}{3}}}$$

d) $a_n = n^2+5$

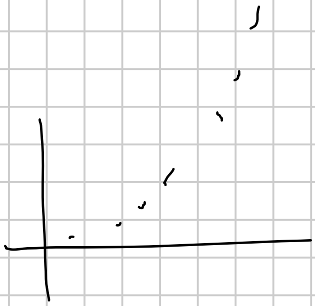
$(a_n) = 6, 9, 14, \dots$

$\lim_{n\to\infty} (n^2+5) = \infty$

divergent

bestimmt-divergent

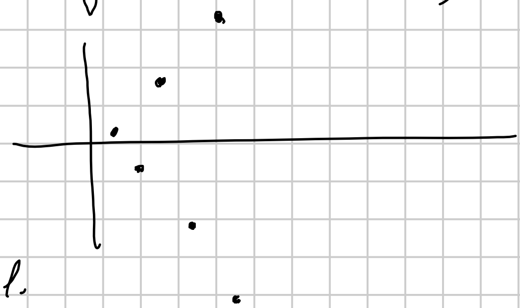
(uneig. GW ist $+\infty$)



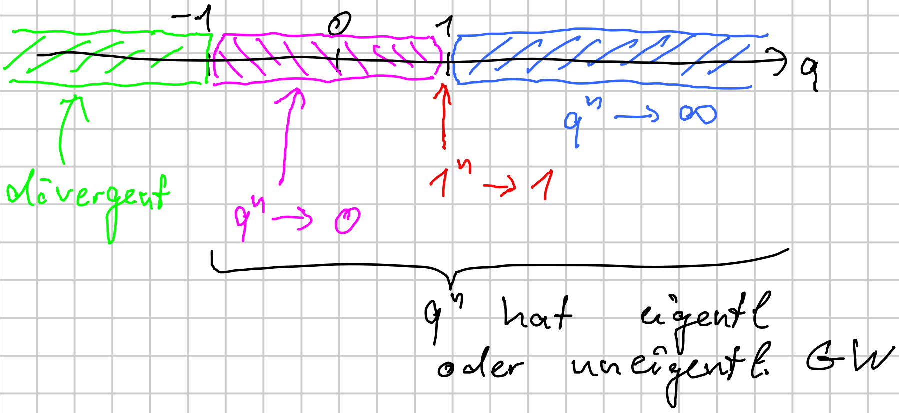
e) $a_n = (-1)^n n^2$

$\lim_{n\to\infty} a_n$ ex. nicht

divergent, kein uneigentl. GW



Geometrische Folge q^n



Zins 1%

1. Jahr $K \cdot 1.01$

2. Jahr $K \cdot 1.01 \cdot 1.01$

$= K \cdot 1.01^2$

⋮

n. Jahr $K \cdot \underbrace{1.01}_{q}{}^n$

Rechnen mit Grenzwerten

Bsp, wieso "$\infty - \infty$" unentscheidbar ist

$$\left.\begin{aligned} a_n &= n \xrightarrow[n\to\infty]{} \infty \\ b_n &= 2n \xrightarrow[n\to\infty]{} \infty \end{aligned}\right\} \quad \begin{aligned} a_n - b_n &= n - 2n = -n \xrightarrow[n\to\infty]{} -\infty \\ b_n - a_n &= 2n - n = n \xrightarrow[n\to\infty]{} +\infty \end{aligned}$$

$$c_n = n+3 \xrightarrow[n\to\infty]{} \infty \longrightarrow a_n - c_n = n-(n+3) = -3 \xrightarrow[n\to\infty]{} -3$$

Bsp zu "Rechnen mit Grenzwerten"

a) $$\lim_{n\to\infty}\left(\frac{1}{n^2}\right) = \lim_{n\to\infty}\left(\frac{1}{n}\cdot\frac{1}{n}\right) = \underbrace{\left(\lim_{n\to\infty}\frac{1}{n}\right)}_{\text{Fund. Nullfolge}}\cdot\left(\lim_{n\to\infty}\frac{1}{n}\right) = 0\cdot 0 = 0$$

b) $$\lim_{n\to\infty}\left(\frac{3\cancel{n^3}}{5\cancel{n^3}}\right) = \left(\frac{\lim_{n\to\infty}(3n^3)}{\lim_{n\to\infty}(5n^3)}\right) = \frac{\infty}{\infty} \quad \text{so nicht entscheidbar}$$

$$\hookrightarrow \lim_{n\to\infty}\left(\frac{3}{5}\right) = \frac{3}{5}$$

c) $$\lim_{n\to\infty}\left(\frac{3n}{5n^4}\right) = \lim_{n\to\infty}\left(\frac{3}{5n^3}\right) = \frac{3}{\infty} = \underline{\underline{0}}$$

d) $$\lim_{n\to\infty}\frac{-2n^2+4n-5}{8n^2-3n+7} \underset{\text{g.P.i.N.}}{=} \lim_{n\to\infty}\frac{\cancel{n^2}\left(-2+\overset{\to 0}{\frac{4}{n}}-\overset{\to 0}{\frac{5}{n^2}}\right)}{\cancel{n^2}\left(8-\underset{\to 0}{\frac{3}{n}}+\underset{\to 0}{\frac{7}{n^2}}\right)} = \frac{-2}{8} = \underline{\underline{-\frac{1}{4}}}$$

e) $$\lim_{n\to\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right) = \lim_{n\to\infty}\frac{1}{n} - \lim_{n\to\infty}\frac{1}{n+1} = 0-0 = \underline{\underline{0}}$$

f) $$\lim_{n\to\infty}\left(\frac{n^2}{n}-\frac{n^2}{n+1}\right) = \infty - \infty \quad \text{unentscheidbar}$$

$$\lim_{n\to\infty} n^2\left(\frac{1}{n}-\frac{1}{n+1}\right) = \infty\cdot 0 \quad \text{immer noch unentscheidbar}$$

Hauptnenner

$$\lim_{n\to\infty}\left(\frac{n^2(n+1)-n^2 n}{n(n+1)}\right) = \lim_{n\to\infty}\left(\frac{\cancel{n^3}+n^2-\cancel{n^3}}{n^2+n}\right) = \lim_{n\to\infty}\frac{n^2}{n^2+n}$$

$$= \lim_{n\to\infty}\frac{1}{1+\frac{1}{n}} = \underline{\underline{1}}$$

Übung

2) $\lim_{n\to\infty}\left(\frac{7n^2-1}{3n^2+2}\right)^2 \underset{\text{g.P.i.N.}}{=} \left(\lim_{n\to\infty}\frac{\cancel{n^2}(7-\frac{1}{n^2})}{\cancel{n^2}(3+\frac{2}{n^2})}\right)^2 = \left(\frac{7}{3}\right)^2 = \frac{49}{9}$

3) $\lim_{n\to\infty}\left(\frac{n^3}{n+1}-\frac{n^3}{n-1}\right) \underset{\text{H.N.}}{=} \lim_{n\to\infty}\left(\frac{n^3(n-1)-n^3(n+1)}{(n+1)(n-1)}\right)$

$= \lim_{n\to\infty}\frac{\cancel{n^4}-n^3-\cancel{n^4}-n^3}{n^2-1} = \lim_{n\to\infty}\frac{-2n^3}{n^2-1} \underset{\text{g.P.i.N.}}{=} \lim_{n\to\infty}\frac{\cancel{n^2}(-2n)}{\cancel{n^2}(1-\frac{1}{n^2})}$

$= \frac{-\infty}{1} = -\infty$ bestimmt divergent

4) $\lim_{k\to\infty} 10^{-k} = \lim_{k\to\infty}\left(\frac{1}{10}\right)^k = \lim_{k\to\infty} 0.1^k = 0$

$\lim_{k\to\infty}\frac{75\cdot 10^k + 6\cdot 10^{2k}}{0.4\cdot 10^{k-3} - 20\cdot 10^{2k-2}} = \lim_{k\to\infty}\left(\frac{\cancel{10^{2k}}(75\cdot 10^{-k} + 6)}{\cancel{10^{2k}}(0.4\cdot 10^{-k-3} - 20\cdot 10^{-2})}\right)$

$= \frac{0+6}{0-20\cdot\frac{1}{100}} = \frac{6}{\left(-\frac{2}{10}\right)} = -30$

Bsp

a) $\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$ | ~~erweitern~~

$\lim_{n\to\infty}\left(\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\left(\sqrt{n+1}+\sqrt{n}\right)}{\left(\sqrt{n+1}+\sqrt{n}\right)}\right) =$ ← 3. Binom. Formel

$\lim_{n\to\infty}\left(\frac{(\cancel{n}+1)-\cancel{n}}{\sqrt{n+1}+\sqrt{n}}\right) = \lim_{n\to\infty}\left(\frac{1}{\sqrt{n+1}+\sqrt{n}}\right) = \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0$

b) $\lim_{n\to\infty}\frac{\sqrt{n^2+1}}{n+3} \underset{\text{g.P.i.N.}}{=} \lim_{n\to\infty}\frac{\sqrt{n^2(1+\frac{1}{n^2})}}{n(1+\frac{3}{n})} = \lim_{n\to\infty}\frac{\cancel{n}\sqrt{1+\frac{1}{n^2}}}{\cancel{n}(1+\frac{3}{n})} = \frac{\sqrt{1}}{1} = 1$

Fixpunkt-Iteration

Bsp: $x+1 = \sin(x) \quad | -1$

$\Leftrightarrow \quad x = \sin(x) - 1 = f(x)$

	$f(\;)$	
$a_1 = 1$	$\sin(1)-1$	$= a_2$
$a_2 = a_2$	$\sin(a_2)-1$	$= a_3$
a_3	$\sin(a_3)-1$	$= a_4$

$$x + 1 = \sin(x) \qquad | \arcsin(\ )$$

$$\arcsin(x+1) = x$$

funktioniert leider nicht

Landau O/Notation

$$n+2 \in \Theta(n) \qquad \text{weil } \lim_{n\to\infty} \left| \frac{n+2}{n} \right| = 1$$

$$\in O(n^2) \qquad \text{weil } \lim_{n\to\infty} \left| \frac{n+2}{n^2} \right| = 0$$