MA1 – 23.10.2019

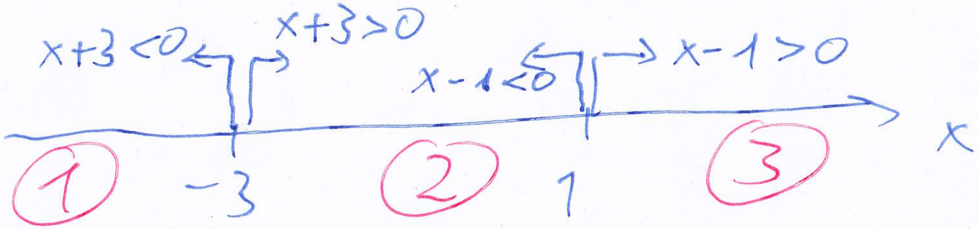
Orga

Ü Di Kg	Ü Mi Zü	Ü Mi Wag	Ü Mi Bag
3.101	3.101	3.108	3.102
35 TN	70 TN	50 TN	50 TN

Ü $\quad \underbrace{|x-1|}_{=0} - 3|\underbrace{x+3}_{=0}| = 1 \qquad (1)$

Umschlagspunkte bestimmen (da wo Argument v |..| gleich 0

1) $x = 1$
2) $x = -3$



Fall 1 $\quad x+3<0 \ \wedge\ x-1<0 \iff x<-3$

$(1): \quad -(x-1) - 3(-(x+3)) = 1$

$\iff -x+1+3x+9 = 1 \iff 2x = -9 \iff$

$\Leftrightarrow \quad x = -\frac{9}{2}$ ✓ passt, denn $x \in (-\infty, -3]$

Fall 2: $x+3>0 \wedge x-1<0 \Leftrightarrow -3<x<1$

$(1): \quad -(x-1) - 3(x+3) = 1$

$\Leftrightarrow \quad -x+1 - 3x - 9 = 1$

$\Leftrightarrow \quad -4x = 9 \Leftrightarrow x = -\frac{9}{4} = -2.25$ ✓

passt in Bereich $x \in (-3,1)$

Fall 3 $x+3>0 \wedge x-1>0 \Leftrightarrow x>1$

$(1): \quad (x-1) - 3(x+3) = 1$

$\Leftrightarrow \quad -2x - 1 - 9 = 1 \Leftrightarrow -2x = 11$

$\Leftrightarrow \quad x = -\frac{11}{2}$ ↯ passt nicht, denn $x \notin (1,\infty)$

$L = \{-\frac{9}{2}, -\frac{9}{4}\}$

Ü Faktorzerlegung

a) $x^2 - 25 = 0 \iff x^2 - 5^2 = 0$

3. Binom Formel
$a^2 - b^2 = (a-b)(a+b)$

$\iff (x-5)\cdot(x+5) = 0$

$\iff \underline{x = 5 \lor x = -5}$

Alternativ $x^2 = 25 \quad | \sqrt{}$

$x = \pm 5$

NR
$(\ldots) = (z + x\cdot z)$
$(1+x)\cdot z$

b) $x \ln(x^2+1) + x^2 \ln(x^2+1) = 0 \qquad D = \mathbb{R}$

$\iff x\,(\underbrace{\ln(x^2+1)}_{z}\cdot 1 + x\,\underbrace{\ln(x^2+1)}_{z}) = 0$

$\iff \underbrace{x}_{a}\cdot(\underbrace{1+x}_{b})\cdot\underbrace{\ln(x^2+1)}_{c} = 0$

$L = \{0, -1\}$

$\iff x = 0 \lor x = -1 \lor \ln(\underbrace{x^2+1}_{=1}) = 0$

$\iff x^2 + 1 = 1 \iff x = 0$

c) $x \ln(x) + x^2 \ln(x^2) = 0$ $\qquad\qquad D = \mathbb{R}^+ = (0,\infty)$

$\Leftrightarrow x \ln(x) + 2x^2 \ln(x) = 0$

$\Leftrightarrow (x + 2x^2) \ln(x) = 0$

$\Leftrightarrow \underbrace{x}_{a} \cdot \underbrace{(1 + 2x)}_{b} \cdot \underbrace{\ln(x)}_{c} = 0$

$\Leftrightarrow x = 0 \vee 1 + 2x = 0 \vee \ln(x) = 0$

$\Leftrightarrow x = 0 \vee x = -\frac{1}{2} \vee x = 1$

$L = \{1\}$ $\quad$ (da $-\frac{1}{2} \notin D$ und $0 \notin D$)

Summen

$$\sum_{k=1}^{n} c = (\underbrace{c+c+\ldots+c}_{n\text{-mal}})$$

c: konstant, d.h. unabhängig von k

$$= (1+1+\ldots+1)\cdot c = nc$$

$$\sum_{k=1}^{20} 5 = 20\cdot 5 = 100$$

ü $$\underline{\left(\sum_{j=1}^{50}(j-1)j\right)} - \underline{\left(\sum_{i=3}^{52} i(i+1)\right)}$$

$$= \cancel{0\cdot 1} + 1\cdot 2 + 2\cdot 3 + \cancel{3\cdot 4} + \ldots + \cancel{49\cdot 50}$$

$$- \cancel{3\cdot 4} - \ldots - \cancel{49\cdot 50} - 50\cdot 51 - 51\cdot 52 - 52\cdot 53$$

$$= 2 + 6 - 50\cdot 51 - 51\cdot 52 - 52\cdot 53$$

$$= \underline{\underline{-7950}}$$

ü separierbare Doppelsummen

a) $$\sum_{k=1}^{2}\sum_{i=1}^{10} k\cdot i = \left(\sum_{k=1}^{2} k\right)\cdot\left(\sum_{i=1}^{10} i\right)$$

$$= (1+2)\,\frac{10\cdot 11}{2} = 165$$

$$\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$$

b) $$\boxed{\sum_{k}\sum_{i}\left(A_{ki} \pm B_{ki}\right) = \sum_{k}\sum_{i} A_{ki} \pm \sum_{k}\sum_{i} B_{ki}}$$

hier:

$$\sum_{k=1}^{3}\sum_{i=1}^{4}\left((k\cdot i)^2 + 5\right) = \sum_{k=1}^{3}\sum_{i=1}^{4} k^2 i^2 + \underbrace{\sum_{k=1}^{3}\underbrace{\sum_{i=1}^{4} 5}_{4\cdot 5}}_{3\cdot 4\cdot 5}$$

$$= \left(\sum_{k=1}^{3} k^2\right)\cdot\left(\sum_{i=1}^{4} i^2\right) + 3\cdot 4\cdot 5$$

$$= (1^2+2^2+3^2)(1+4+9+16) + 60 = 480$$

Binomialkoeffizient

$$\binom{n}{n-k} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k} \qquad (\text{Symmetrie } k \leftrightarrow n-k)$$

Bsp: $\binom{100}{98} = \binom{100}{2}$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{\overbrace{n\cdot\ldots\cdot(n-k+1)}^{k\text{ Terme}}\cdot\overbrace{\cancel{(n-k)}\cdot\ldots\cdot 1}^{(n-k)!}}{k!\;\cancel{(n-k)}!}$$

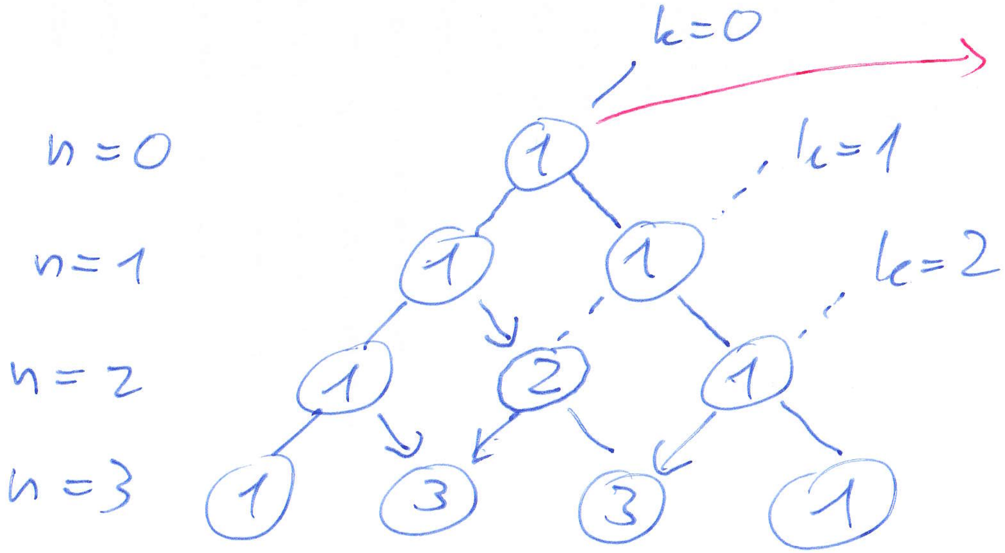
$$= \frac{k \text{ Terme von } n \text{ abwärts bis } (n-k+1)}{k \text{ Terme von } k \text{ abwärts bis } 1}$$

Bsp: $\binom{100}{2} = \frac{100\cdot 99}{2\cdot 1} = \frac{9900}{2} = 4950$

$\binom{100}{50}$ => viel Arbeit

Pascal'sches Dreieck

Wert von $\binom{n}{k} = \binom{0}{0}$



n\k	0	1	2	3	4	5	6	7
0	1							
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	6	4	1			
5	1	5	10	10	5	1		
6	1	6	15	20	15	6	1	
7	1	7	21	35	35	21	7	1

$$(a+b)^7 = 1\cdot b^7 + 7ab^6 + 21a^2b^5 + 35a^3b^4 + 35a^4b^3 + 21a^5b^2$$
$$+ 7a^6b + 1\cdot a^7$$

Vereinfachen

a) $$\frac{(n+1)!}{(n-1)!} = \frac{(n+1)\cdot n\,\cancel{(n-1)!}}{\cancel{(n-1)!}} = (n+1)\,n$$

b) $$\frac{1}{n!} + \frac{1}{(n-1)!}$$ | auf $\underbrace{\text{Hauptnenner}}_{n!}$ bringen

$$= \frac{1}{n!} + \frac{1}{(n-1)!}\,\frac{n}{n}$$

$$= \frac{1 + n}{n!}$$