V MA1 13.11.2019

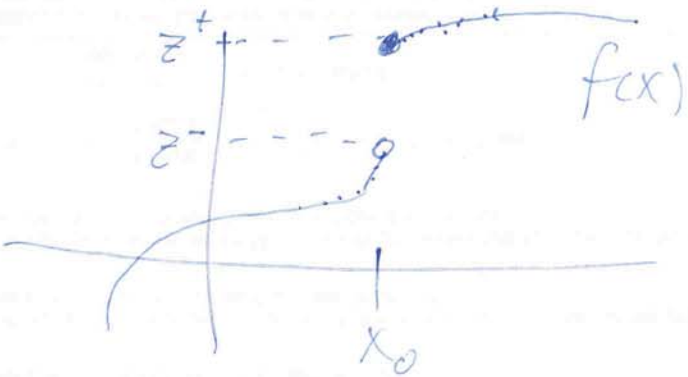
Orga HIP-Woche 18. - 22.11 keine V+Ü

Zu Terminen P: Beate Breiderhoff

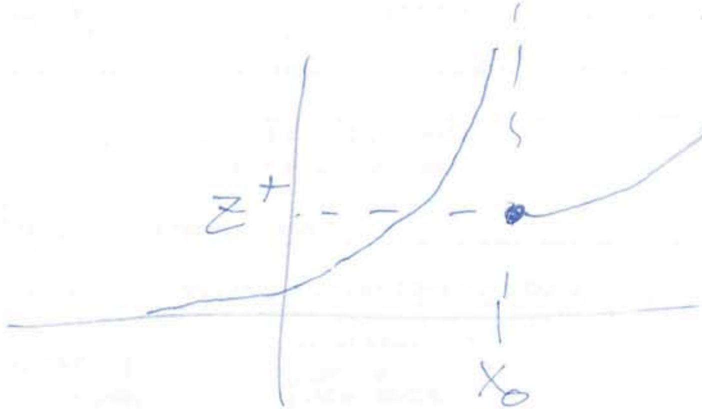
Mi 11^{30} - 12^{30} Sprechstunde in 0.5xx

Ü wichtig!

Bsp zu links/rechtsseitiger GW



f hat bei x_0 keinen GW



kein GW

da $z^- = +\infty \neq z^+$

Bsp

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} \overset{(*)}{=} x+1 & \text{für } x \in \mathbb{R}^+ \setminus \{1\} \\ -\frac{1}{x} & \text{f. } x < 0 \end{cases}$$

$\lim\limits_{x \to -\infty} f(x) = 0$ denn $f(x_n) \xrightarrow[n\to\infty]{} -\infty$ gilt $\lim\limits_{n\to\infty} f(x_n) = \lim\limits_{n\to\infty}\left(-\frac{1}{x_n}\right) = -\frac{1}{\infty} = \underline{\underline{0}}$

$\lim\limits_{x \to 0-} f(x) = +\infty$ denn für $(x_n) \xrightarrow[n\to\infty]{} 0-$ gilt:

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}\left(-\frac{1}{x_n}\right) = -\frac{1}{0^-} = -(-\infty) = +\infty$$

$\lim\limits_{x \to 0+} f(x) = 1$ denn für $(x_n) \xrightarrow[n\to\infty]{} 0+$ gilt:

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}\left(\frac{x_n^2-1}{x_n-1}\right) = \frac{0-1}{0-1} = \underline{\underline{1}}$$

$\lim\limits_{x \to 0} f(x)$ nicht definiert $(z^- \neq z^+)$

$\lim\limits_{x \to 1} f(x) = 2$ denn für $(x_n) \xrightarrow[n\to\infty]{} 1$ gilt

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}\left(\frac{x_n^2-1}{x_n-1}\right) \overset{(*)}{=} \lim_{n\to\infty}(x_n+1) = 1+1 = \underline{\underline{2}}$$

ü

a) $\lim\limits_{x\to 1}\left[(x+1)\cos(x-1)+\frac{\sin(x-1)}{x+1}\right]$

$$2\cdot\cos 0 + \frac{\sin 0}{2} = \underline{\underline{2}}$$

b) $\lim\limits_{x\to 1}\left[\frac{x+2}{x-1}-\frac{x^2+2x+3}{x^2-1}\right]$

Einsetzen führt auf $\frac{3}{0}-\frac{6}{0}=\infty-\infty$ ↯

Nach Nr. 5: Auf Hauptnenner $\left[x^2-1=\underbrace{(x-1)(x+1)}_{HN}\right]$

$$=\lim_{x\to 1}\left[\frac{(x+2)(x+1)}{(x-1)(x+1)}-\frac{x^2+2x+3}{(x-1)(x+1)}\right]$$

$$=\lim_{x\to 1}\frac{\cancel{x^2}+2x+x+2-(\cancel{x^2}+2x+3)}{(x-1)(x+1)}=\lim_{x\to 1}\frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$$

$$=\lim_{x\to 1}\frac{1}{x+1}=\frac{1}{1+1}=\underline{\underline{\frac{1}{2}}}$$

Bsp 1) Was ist $\lim\limits_{x\to 0} \frac{\sin x}{x}$?

Formelsammlung $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - + \dots$

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\cancel{x} - \frac{x^{\cancel{3}\,2}}{3!} + \frac{x^{\cancel{5}\,4}}{5!} - + \dots}{\cancel{x}} \quad \cancel{-\frac{x^2}{3!} + \frac{x^4}{5!} - + \dots}$$

$$= \lim_{x\to 0}\frac{1 - \frac{x^2}{3!} + \frac{x^4}{5!} - + \dots}{1} = \underline{\underline{1}}$$

2) $$\lim_{x\to(-2)-}\frac{x^2+2x+2}{x+2} = \frac{(-2)^2+2\cdot(-2)+2}{0-} = \frac{2}{0-} = -\infty$$

Für jede Folge $(x_n) \xrightarrow[n\to\infty]{} (-2)-$ gilt

$$\cancel{\lim_{x\to(-2)-}}\ \lim_{n\to\infty}\frac{x_n^2+2x_n+2}{x_n+2} = \frac{(-2)^2+2\cdot(-2)+2}{((-2)-)+2}$$

$$= \frac{2}{0-}$$

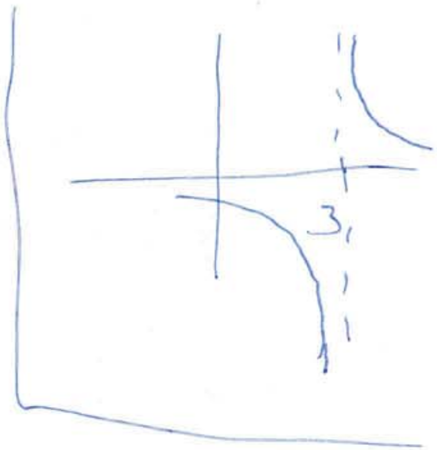
3) $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - x^2}{h} = \lim\limits_{h\to 0} \dfrac{x^2+2xh+h^2-x^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{2x\cancel{h} + h^{\cancel{2}1}}{\cancel{h}} = \lim\limits_{h\to 0} (2x+h) = \underline{\underline{2x}}$

Ü a) $\lim\limits_{x\to 3+} \dfrac{x^2-3x+5}{x-3} = \dfrac{(3+)\cdot 3 - 3\cdot 3 + 5}{3^+ - 3} = \dfrac{5}{0+} = +\infty$

b) $\lim\limits_{x\to 3-} \dfrac{x^2-3x+5}{x-3} = \dfrac{5}{0-} = -\infty$

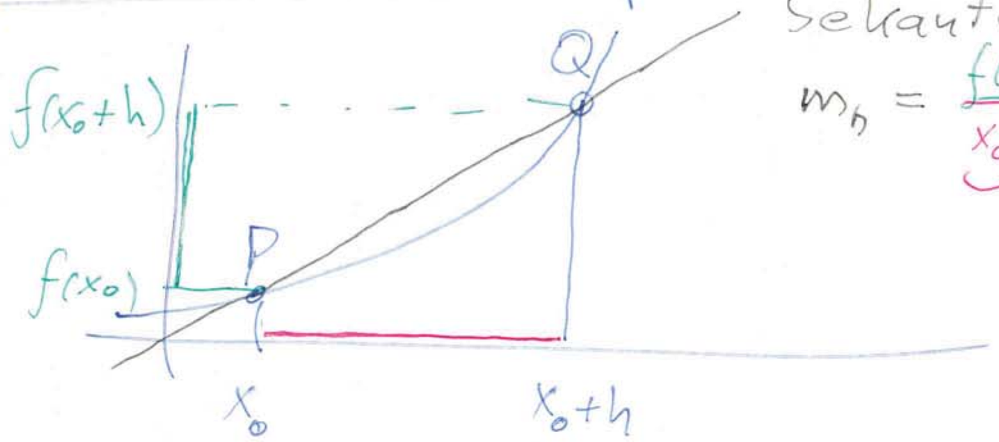
$\Big\}$ $\lim\limits_{x\to 3} f(x)$ ex. nicht



c) $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{x-3} = \lim\limits_{x\to 3} \dfrac{x^2-2\cdot 3x+3^2}{x-3}$

$= \lim\limits_{x\to 3} \dfrac{(x-3)^2}{x-3} = \lim\limits_{x\to 3} (x-3) = \underline{\underline{0}}$

Differentialrechnung



Sekante mit Steigung

$$m_h = \frac{f(x_0+h) - f(x_0)}{\underbrace{x_0+h-x_0}_{h}}$$

Steigung im Punkte P: lasse Q zu P wandern

$$\boxed{\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0)}$$

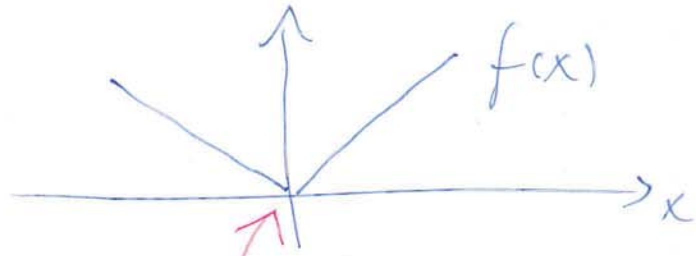
Bsp: $f(x) = x^2$

$$f'(x_0) = \lim_{h\to 0} \frac{(x_0+h)^2 - x_0^2}{h} = \lim_{h\to 0} \frac{2x_0h + h^2}{h} = \underline{\underline{2x_0}}$$

$\Rightarrow f'(x) = 2x$

Nicht jede Fkt diff. bar

$f(x) = |x|$, $x_0 = 0$



Funktion hat Knick bei $x_0 = 0$

Bsp. Quotientenregel

$$f(x) = \frac{u(x)}{v(x)} = \frac{x^2}{x+1} \qquad \Big| \quad u' = 2x, \quad v' = (x+1)' = 1$$

$$f'(x) = \frac{\underbrace{(x+1)}_{v}\cdot 2x - x^2 \cdot \underbrace{1}_{v'}}{\underbrace{(x+1)^2}_{v^2}} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$$

<u>Übung</u> Sei $f(x) = \frac{1}{x}$. Berechnen Sie die 1., 2., 3. Ableitung

$$f(x) = x^n$$
$$f'(x) = n\,x^{n-1}$$
$$\forall n \in \mathbb{Z}$$

<u>1. Lösungsweg</u>

$$f(x) = x^{-1}$$
$$f'(x) = (-1)\,x^{-2} = -x^{-2} = -\frac{1}{x^2}$$
$$f''(x) = (-1)(-2)\,x^{-3} = 2x^{-3} = \frac{2}{x^3}$$
$$f'''(x) = -6\,x^{-4}$$

<u>2. Lösungsweg</u>

$$\left.\begin{array}{ll} u = 1 & u' = 0 \\ v = x & v' = 1 \end{array}\right\} \quad f'(x) = \frac{x \cdot 0 - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$$

$$f'(x) = -\frac{1}{x^2} : \left[\begin{array}{ll} u = 1 & u' = 0 \\ v = x^2 & v' = 2x \end{array}\right] \quad f''(x) = \frac{x^2 \cdot 0 - 2x \cdot 1}{x^4} = +\frac{2}{x^3}$$

Kettenregel Was ist $\left[g(\underbrace{f(x)}_{u})\right]'$?

Antwort $g'(u)\big|_{u=f(x)} \cdot f'(x)$

Bsp $h(x) = \Big(\underbrace{\frac{x^2}{x+1}}_{u}\Big)^2$

$h(x) = g(f(x))$

mit $g(u) = u^2$, $g'(u) = 2u$

$f(x) = \frac{x^2}{x+1}$, $f'(x) = \frac{x^2+2x}{(x+1)^2}$

s.o. Bsp Quotientenregel

$$h'(x) = 2u\Big|_{u=f(x)} \cdot \frac{x^2+2x}{(x+1)^2} = 2\,\frac{x^2}{x+1} \cdot \frac{(x^2+2x)}{(x+1)^2}$$

$$= \frac{2x^2(x^2+2x)}{(x+1)^3}$$

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Ü Kettenregel b) c)

b) $g(x) = \sin(x^3) = h(d(x))$ mit $h(u) = \sin(u)$, $h'(u) = \cos(u)$
$d(x) = x^3$ $d'(x) = 3x^2$

$g'(x) = \cos(x^3) \cdot 3x^2$

c) $h(x) = e^{\sin(x^2)}$

1. Teilaufg. Was ist $(\sin(x^2))' = \cos(x^2) \cdot 2x$
2. Teilaufg. $e^{\sin(x^2)} = g(f(x))$ mit $g(u) = e^u$ $g' = e^u$

$f(x) = \sin(x^2)$ $f' = \cos(x^2) \cdot 2x$

1. Teilaufg.

$$\boxed{h'(x) = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x}$$