# Grenzwerte 1

Mittwoch, 25. November 2020 08:17

entscheidbare Fälle

Entscheidbare Fälle ²

1) sei: $a_n = 5$, $b_n = n \xrightarrow[n\to\infty]{} \infty$

$\lim\limits_{n\to\infty}(5+n^2) = 5+\infty = \infty$

analog: $5-\infty = -\infty$

2) $\lim\limits_{n\to\infty}\left(\frac{3}{n^2}\right) = \frac{3}{\infty} = 0$

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Unentscheidbare Fälle

1) $0\cdot\infty = \,?$ C

a) $\lim\limits_{n\to\infty}\left(\underbrace{\tfrac{1}{n}}_{\downarrow\,0}\cdot\underbrace{n}_{\downarrow\,\infty}\right) = \lim\limits_{n\to\infty}(1) = \underline{\underline{1}}$

Termumformungen

b) $\lim\limits_{n\to\infty}\left(\frac{1}{n^{\cancel{2}}}\cdot\cancel{n}\right) = \lim\limits_{n\to\infty}\left(\frac{1}{n}\right) = \underline{\underline{0}}$

c) $\lim\limits_{n\to\infty}\left(\frac{1}{n}\cdot n^4\right) = \lim\limits_{n\to\infty}(n^3) = \underline{\underline{\infty}}$

2) $\infty - \infty = \,?$

$2n - n = +n \xrightarrow[n\to\infty]{} \underline{\underline{+\infty}}$

$n-(n+3) = -3 \xrightarrow[n\to\infty]{} \underline{\underline{-3}}$

$n - 3n = -2n \xrightarrow[n\to\infty]{} \underline{\underline{-\infty}}$

# Grenzwerte2

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1) $\lim\limits_{n\to\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) = 0 - 0 = \underline{\underline{0}}$

2) $\lim\limits_{n\to\infty} n^2\left(\frac{1}{n} - \frac{1}{n+1}\right) = \underline{\underline{\lim\limits_{n\to\infty}\left(\frac{n^2}{n} - \frac{n^2}{n+1}\right)}}$

$= \infty - \infty$ unentscheidbar

$= \frac{1}{n} \lim\limits_{n\to\infty}\left(\frac{n^2}{1} - \frac{n^2}{1+\frac{1}{n}}\right)$

$\cancel{\frac{n^2}{n} = \frac{n^2+1}{n+1}}$

Hauptnenner

$= \lim\limits_{n\to\infty}\left(\frac{n^2(n+1) - n^2\cdot n}{n(n+1)}\right)$

$= \lim\limits_{n\to\infty}\left(\frac{\cancel{n^3} + n^2 - \cancel{n^3}}{n^2+n}\right)$

z.P.i.N

$= \lim\limits_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right) = \underline{\underline{1}}$

# Ü Grenzwerte

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2) $\lim\limits_{n\to\infty}\left(\frac{7n^2-1}{3n^2+2}\right)^2 = \left(\lim\limits_{n\to\infty}\frac{7n^2-1}{3n^2+2}\right)^2$

$= \left(\lim\limits_{n\to\infty}\left(\frac{7-\frac{1}{n^2}}{3+\frac{2}{n^2}}\right)\right)^2 = \left(\frac{7-0}{3+0}\right)^2 = \left(\frac{7}{3}\right)^2$

$= \underline{\underline{\frac{49}{9}}}$

3) $\lim\limits_{n\to\infty}\left(\frac{n^3}{n+1} - \frac{n^3}{n-1}\right) \underset{\text{H.N.}}{=} \lim\limits_{n\to\infty}\frac{n^3(n-1)-n^3(n+1)}{(n+1)(n-1)} =$

$\lim\limits_{n\to\infty}\left(\frac{\cancel{n^4}-n^3-\cancel{n^4}-n^3}{n^2-1}\right) = \lim\limits_{n\to\infty}\frac{-2n^3}{n^2-1} =$

$\lim\limits_{n\to\infty}\frac{\cancel{n^2}(-2n)}{\cancel{n^2}\left(1-\frac{1}{n^2}\right)} = \underline{\underline{-\infty}}$ bestimmt-divergent

4) $\lim\limits_{k\to\infty}\frac{75\cdot 10^k + 6\cdot 10^{2k}}{0.4\cdot 10^{k-3} - 20\cdot 10^{2k-2}}$

$10^{2k-2}$
$= 10^{2k}\cdot 10^{-2}$
$= 10^{2k}\cdot\frac{1}{100}$

$= \lim\limits_{k\to\infty}\frac{10^{2k}(75\cdot \underbrace{10^{-k}}_{\to 0} + 6)}{10^{2k}(0.4\cdot \underbrace{10^{-k-3}}_{\to 0} - 20\cdot\frac{1}{100})}$

$= \frac{0+6}{0-\frac{20}{100}} = \underline{\underline{-30}}$

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Bsp $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+1}}{n+3} \underset{\text{g.P.i.N.}}{=} \lim\limits_{n\to\infty}\frac{\sqrt{n^2\left(1+\frac{1}{n^2}\right)}}{n\left(1+\frac{3}{n}\right)}$

$= \lim\limits_{n\to\infty}\frac{\cancel{n}\sqrt{1+\frac{1}{n^2}}}{\cancel{n}\left(1+\frac{3}{n}\right)} = \underline{\underline{1}}$

# Fixpunkt

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Bsp

$$\boxed{x + 1 = \sin(x)}$$

$$x = \underbrace{\sin(x) - 1}_{f(x)} \qquad \text{F.P.-Form}$$

$$x \to a_{n+1} \qquad f(x) \to a_n$$

| $n$ | $a_n$ | $f(a_n) = a_{n+1}$ |
|---|---|---|
| 1 | $a_1 = 1$ | $a_2 = \sin(a_1) - 1$ |
| 2 | $a_2$ | $a_3 = \sin(a_2) - 1$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

Es gibt viele (unendlich viele) Möglichkeiten nach x aufzulösen

$$x + 1 = \sin(x) \qquad | \arcsin()$$

$$\underbrace{\arcsin(x+1)}_{f(x)} = x$$

divergiert leider

$$x + 1 = \sin(x) \qquad | +2x$$

$$3x + 1 = \sin(x) + 2x$$

$$x = \underbrace{\frac{\sin(x) + 2x - 1}{3}}_{f(x)}$$

# O-Notation

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(1) $2n^3 - n^2$ | $\Theta(n^3)$ $\qquad 100n^3 + n^4$

(2) $7n^5 + 26n^6$ | $\Theta(n^6)$ $\quad$ ||

(3) $n + 3n^2 - 2n\log(n)$ | $\Theta(n^2)$

(4) $\dfrac{n^4 + n^2}{n+5}$ | $\Theta(n^3)$, denn $\lim\limits_{n\to\infty} \dfrac{n^4+n^2}{n^3(n+5)} = \ldots = 1$

(5) $\dfrac{n^4+n^2}{n+5} - n^3$ | $\Theta(n^2)$

$$= \frac{n^4 + n^2 - n^3(n+5)}{(n+5)} = \frac{n^2 - 5n^3}{n+5}$$