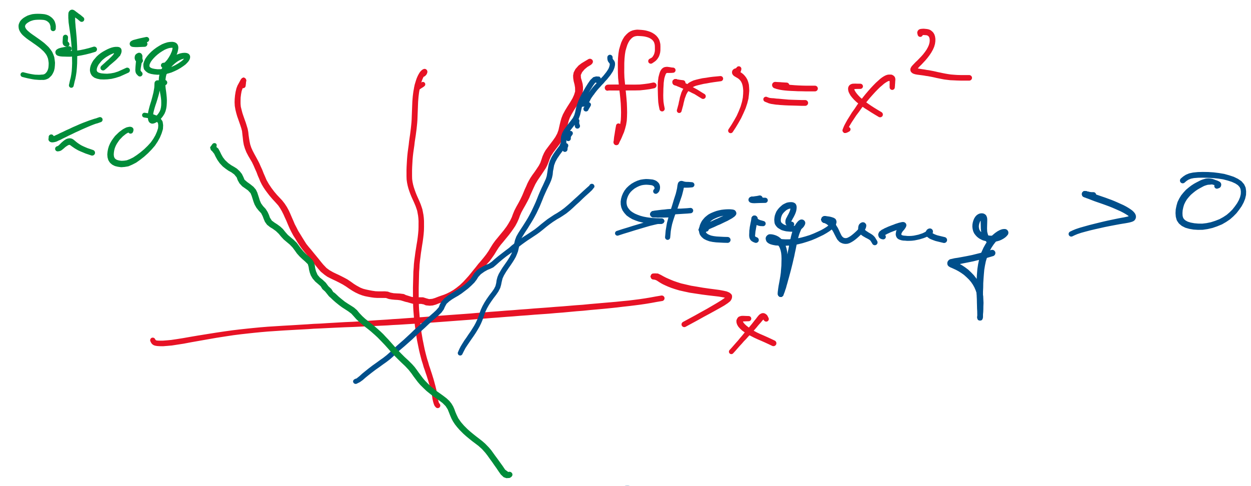
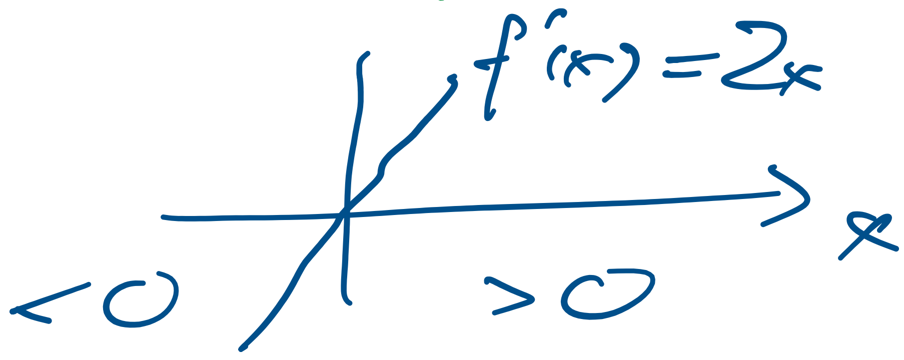
Eigens. diff. Fkt.

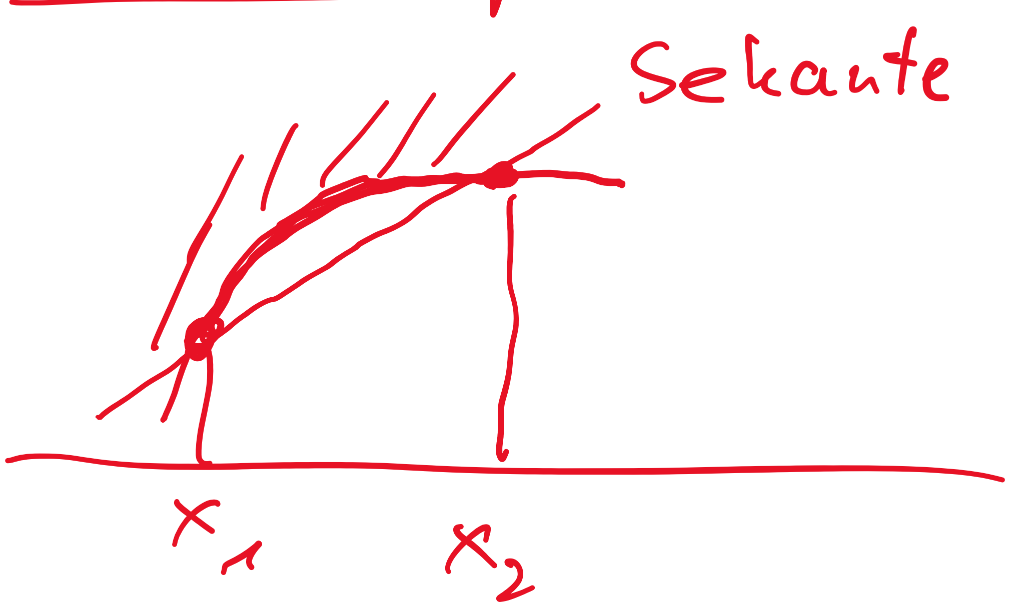
Mittwoch, 16. Dezember 2020 08:53

Bsp Monotonie



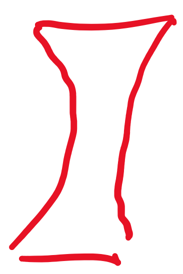


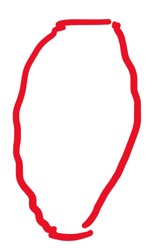
Krümmungsverhalten



konkav | konvex

Optik Linse:





Merkspruch

Ist der Bauch konkav, war die Mama brav

ist der Bauch konvex, hatte Mama ...

Konvexität wichtig f. Optimierung

→ Lokales Min. ist auch global

allg. Funktion

konvexe Fkt

lok = global

Beispiel Diff

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$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 0$

$\Leftrightarrow \quad 3x \cdot (x-2) = 0$

$\Leftrightarrow \quad x_1 = 0 \quad \vee \quad x_2 = 2$

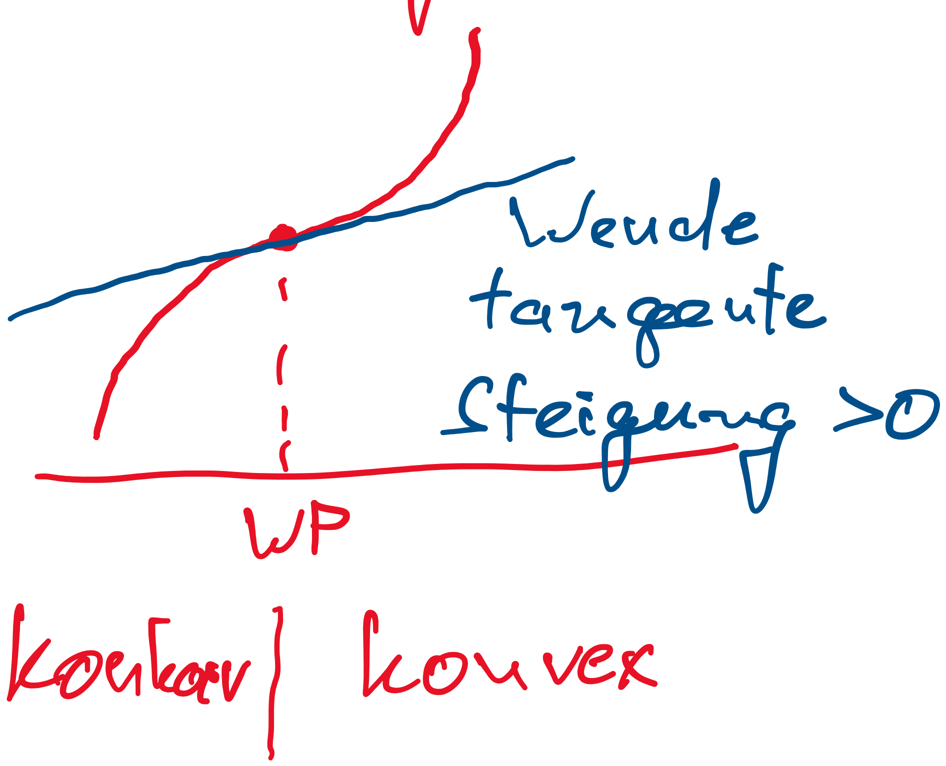
sind Kandidaten für Extrema

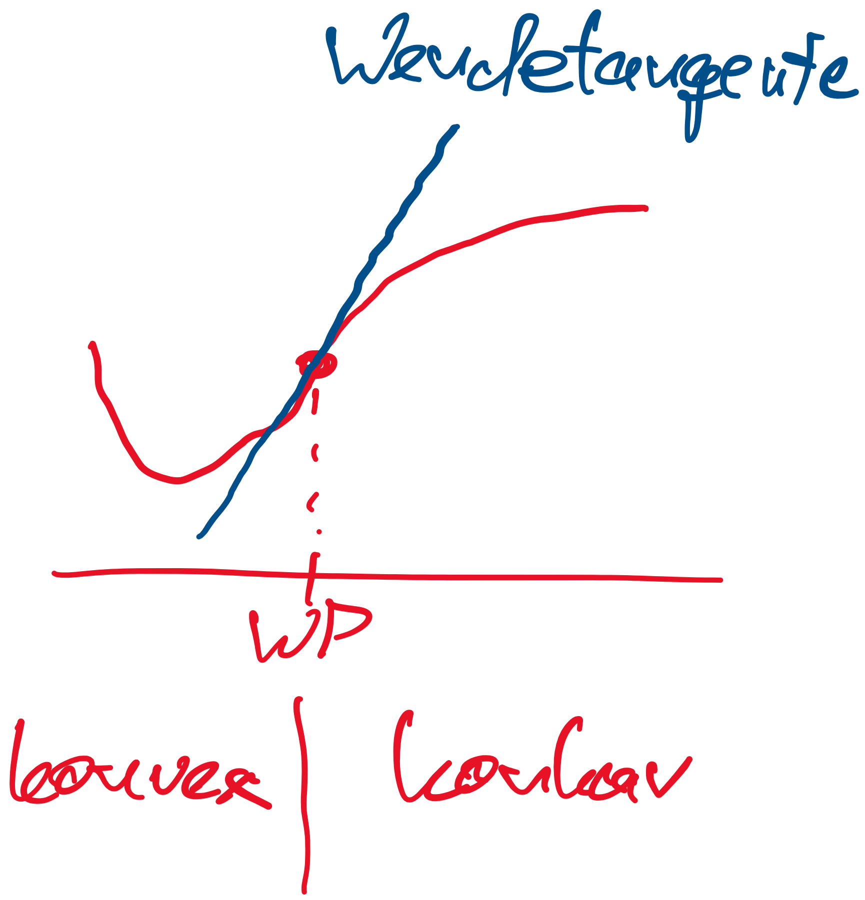
$f''(x) = 6x - 6$

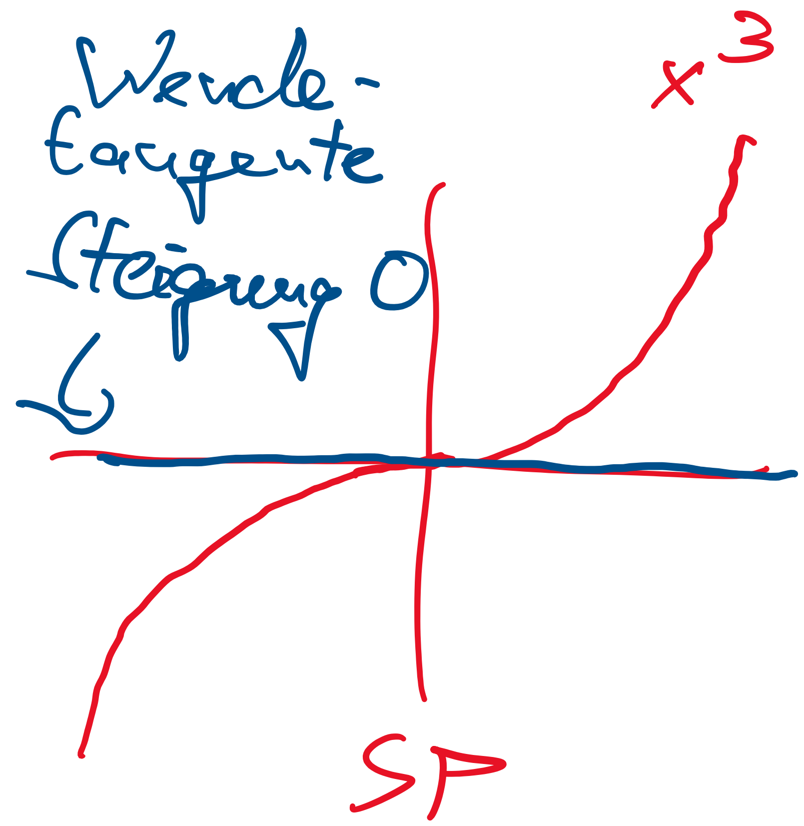
$f''(0) = -6 < 0 \Rightarrow x = 0$ ist Max.

$f''(2) = 12 - 6 = 6 > 0 \Rightarrow x = 2$ " Min.

Wendepunkt







Übung Diff

Mittwoch, 16. Dezember 2020 09:45

$$f(x) = \underbrace{x}_{u}\ \underbrace{(x-3)^2}_{v}$$

$\boxed{\text{Nullstellen } x=0,\ x=3}$

$u' = 1$
$v' = 2(x-3)$

$$f'(x) \underset{\uparrow}{=} 1\cdot(x-3)^2 + x\cdot 2(x-3)$$

Prod. regel

$$= (x-3)\left[(x-3) + 2x\right]$$

$$= (x-3)(3x-3)$$

$$= 3(x-3)\cdot(x-1) = 0$$

$$\Leftrightarrow \quad x_1 = 3 \ \vee \ x_2 = 1$$

sind Kandidaten für Extrema

$$f'(x) = 3x^2 - 12x + 9$$

$$f''(x) = 6x - 12$$

$$f'''(x) = 6$$

Extrema: $f''(3) = 6 > 0 \Rightarrow x = 3$ Min.

$f''(1) = -6 < 0 \Rightarrow x = 1$ Max

Wendepunkt: $f''(x) = 0$

$$\Leftrightarrow \quad 6x - 12 = 0 \Leftrightarrow x = 2$$

$$f'''(2) = 6 \neq 0$$

$\Rightarrow$ $\boxed{x=2 \text{ ist WP}}$

Wendetangente: $f'(2) = -3$

$$f(2) = 2\cdot(2-3)^2 = 2$$

$$w(x) = f(2) + f'(2)(x-2)$$

$$= 2 + (-3)(x-2) = \underline{\underline{-3x+8}}$$