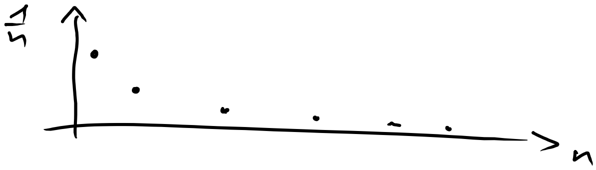
Bsp zu Grenzwert $(n \to \infty)$

1) $a_n = \frac{1}{n}$

$\lim_{n\to\infty} a_n = 0$ "Nullfolge"

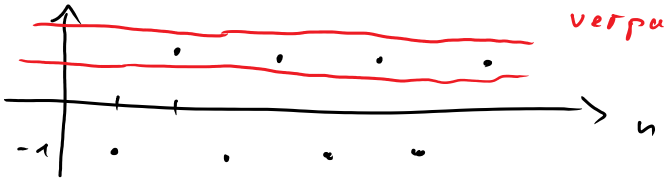
$a_n = \frac{1}{n} \xrightarrow[n\to\infty]{} 0$



2) $a_n = (-1)^n$

$\lim_{n\to\infty} a_n$: ex. nicht

divergent



3) $a_n = \frac{2n-1}{3n}$

$(a_n) = \frac{1}{3}, \frac{3}{6}, \frac{5}{9}, \ldots$ | $\xrightarrow[n\to\infty]{} 0$ "Nullfolge"

$\lim_{n\to\infty} \frac{2n-1}{3n} = \lim_{n\to\infty} \frac{\cancel{n}(2-\frac{1}{n})}{\cancel{n}\cdot 3} = \frac{2-0}{3} = \frac{2}{3}$

~~$n = 1000$: $a_n = \frac{1999}{3000} \approx \frac{2}{3}$~~

konvergent mit Grenzwert $\frac{2}{3}$

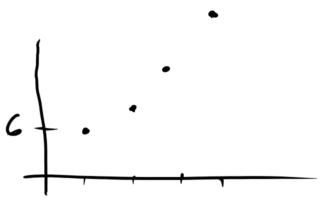
4) $a_n = n^2 + 5$

$(a_n) = 6, 9, 14 \ldots$

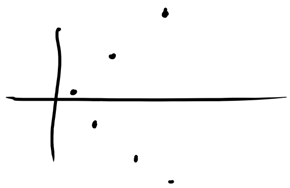
divergent

bestimmt-divergent

(uneigentl. Grenzwert ist $+\infty$) $\lim_{n\to\infty} (n^2+5) = \infty$



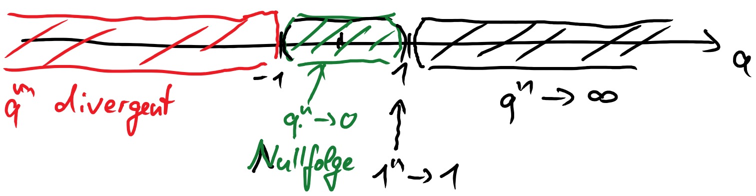
5) $a_n = (-1)^n n^2$



$\lim_{n\to\infty} a_n$ ex. nicht

divergent, kein uneigentl. GW

6) Geometrische Folge q^n, $q \in \mathbb{R}$



q^n hat eigentl. oder uneigentl. GW

Zins 1%, Kapital K

1. Jahr $K \cdot 1.01$

2. Jahr $K \cdot (1.01)^2$

$\vdots$

n. Jahr $K \underbrace{(1.01)^n}_{q}$

Warum ist $\infty - \infty$ unentscheidbar?

$\left.\begin{array}{l} a_n = n \to \infty \\ b_n = 2n \to \infty \end{array}\right\}$ $a_n - b_n = n - 2n = -n \xrightarrow[n\to\infty]{} -\infty$ best. div.

$\left.\begin{array}{l} a_n = n \to \infty \\ b_n = n+5 \to \infty \end{array}\right\}$ $a_n - b_n = n - (n+5) \xrightarrow[n\to\infty]{} -5$ also konv.

$\left.\begin{array}{l} a_n = 2n \to \infty \\ b_n = n \to \infty \end{array}\right\}$ $a_n - b_n = 2n - n = n \xrightarrow[n\to\infty]{} +\infty$

Warum ist $0 \cdot \infty$ unentscheidbar?

$\left.\begin{array}{l} a_n = \frac{1}{n} \to 0 \\ b_n = n \to \infty \end{array}\right\}$ $a_n \cdot b_n = \frac{1}{n} \cdot n = 1$ also konv.

$\left.\begin{array}{l} a_n = \frac{1}{n} \to 0 \\ b_n = n^2 \to \infty \end{array}\right\}$ $a_n \cdot b_n = \frac{1}{n} n^2 = n \to \infty$, also best. div.

Bsp. "Rechnen mit Grenzwerten"

1) $\lim\limits_{n\to\infty}\left(\frac{1}{n^2}\right) = \lim\limits_{n\to\infty}\left(\frac{1}{n}\cdot\frac{1}{n}\right) = \left(\lim\limits_{n\to\infty}\frac{1}{n}\right)\cdot\left(\lim\limits_{n\to\infty}\frac{1}{n}\right) = 0\cdot 0 = \underline{\underline{0}}$

2) $\lim\limits_{n\to\infty}\left(\frac{3n^3}{5n^3}\right) = \frac{\lim\limits_{n\to\infty} 3n^3}{\lim\limits_{n\to\infty} 5n^3} = \frac{\infty}{\infty}$ ↯ nicht entscheidbar

$$\boxed{\lim_{n\to\infty}\left(\frac{3\cancel{n^3}}{5\cancel{n^3}}\right) = \underline{\underline{\frac{3}{5}}}}$$

3) $\lim\limits_{n\to\infty}\left(\frac{-2n^2+4n+5}{8n^2-3n+7}\right) = \lim\limits_{n\to\infty}\frac{\cancel{n^2}\left(-2+\frac{4}{n}+\frac{5}{n^2}\right)}{\cancel{n^2}\left(8-\frac{3}{n}+\frac{7}{n^2}\right)} = \frac{-2+0+0}{8+0+0} = \frac{-2}{8} = \underline{\underline{-\frac{1}{4}}}$

g.P.i.N.

4) $\lim\limits_{n\to\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right) = 0 - 0 = 0$

5) $\lim\limits_{n\to\infty}\left(\frac{n^2}{n}-\frac{n^2}{n+1}\right) = \infty - \infty$ ↯ unentscheidbar

$\lim\limits_{n\to\infty} n^2\left(\frac{1}{n}-\frac{1}{n+1}\right) \underset{(4)}{=} \infty\cdot 0$ ↯ "

3. Versuch: auf Hauptnenner bringen

$\lim\limits_{n\to\infty}\frac{n^2(n+1)-n^2\cdot n}{n(n+1)} = \lim\limits_{n\to\infty}\frac{\cancel{n^3}+n^2-\cancel{n^3}}{n^2+n} = \lim\limits_{n\to\infty}\frac{\cancel{n^2}\cdot 1}{\cancel{n^2}\left(1+\frac{1}{n}\right)} = \underline{\underline{1}}$

Mittwoch, 27. Oktober 2021 10:49

Übung Grenzwert

2) $\lim_{n\to\infty}\left(\frac{7n^2-1}{3n^2+2}\right)^2 = \left(\lim_{n\to\infty}\frac{7n^2-1}{3n^2+2}\right)^2 = \left(\lim_{n\to\infty}\frac{\cancel{n^2}(7-\overbrace{\tfrac{1}{n^2}}^{\to 0})}{\cancel{n^2}(3+\underbrace{\tfrac{2}{n^2}}_{\to 0})}\right)^2$

$= \left(\frac{7}{3}\right)^2 = \frac{49}{9}$

3) $\lim_{n\to\infty}\left(\frac{n^3}{n+1}-\frac{n^3}{n-1}\right) \underset{\text{Hauptnenner}}{=} \lim_{n\to\infty}\frac{n^3(n-1)-n^3(n+1)}{(n+1)(n-1)} = \lim_{n\to\infty}\frac{\cancel{n^4}-n^3-\cancel{n^4}-n^3}{n^2-1}$

$= \lim_{n\to\infty}\frac{-2n^3}{n^2-1} \underset{\text{g.P.i. N.}}{=} \lim_{n\to\infty}\frac{\cancel{n^2}(-2n)}{\cancel{n^2}(1-\underbrace{\tfrac{1}{n^2}}_{0})} = \lim_{n\to\infty}\left(\frac{-2n}{1}\right) = -\infty$

4) $\lim_{k\to\infty}10^{-k} = \lim_{k\to\infty}\left(\frac{1}{10}\right)^k = \lim_{n\to\infty}(0.1)^k = 0$ (geomtr. Folge mit $q=0.1$)

$\lim_{k\to\infty}\frac{75\cdot 10^k+6\cdot 10^{2k}}{0.4\cdot 10^{k-3}-20\cdot 10^{2k-2}} \underset{\text{g.P.i.N.}}{=} \lim_{k\to\infty}\frac{\cancel{10^{2k}}(75\cdot 10^{k-2k}+6\cdot 10^{2k-2k})}{\cancel{10^{2k}}(0.4\cdot 10^{k-3-2k}-20\cdot 10^{2k-2-2k})}$

$= \lim_{k\to\infty}\frac{75\cdot\overbrace{\cancel{10^{-k}}}^{\to 0}+6}{0.4\cdot\underbrace{\cancel{10^{-k}}}_{\to 0}\cdot 10^{-3}-20\cdot 10^{-2}} = \frac{6}{\left(-\frac{20}{100}\right)} = -\frac{60}{2} = -30$

Bsp Fixpunkt iteration

Bestimme eine Lsg der Gleichung

$$x + 1 = \sin(x) \quad | -1$$

transzendente Gleichung, nicht direkt lösbar

Numerische Lösung über F.P.-Iteration

$$\Leftrightarrow \underbrace{x}_{a_{n+1}} = \sin(\underbrace{x}_{a_n}) - 1 = f(x)$$

n	a_n	$a_{n+1} = f(a_n)$
1	1	$\sin(1) - 1 = a_2 = -1.841$
2	-1.841	$\sin(-1.841) - 1 = a_3 = -1.963$
3	-1.963	$\ldots \quad a_4 = -1.923$
$\vdots$		$\vdots$

$$a_{10} \approx x \approx -1.934$$

<u>O - Notation</u>

Ist $a_n = 2n^3 - n^2 \in \mathcal{O}(n^2)$?

$$\left|\frac{a_n}{n^2}\right| = \left|\frac{2n^3 - n^2}{n^2}\right| = \left|\frac{2n-1}{1}\right| \xrightarrow[n\to\infty]{} \infty$$

nicht beschränkt

Also: **Nein**