

Orga Ü 1.11. Feiertag $\Rightarrow$ ü wk wird verlegt
auf Mi, 2.11., 13⁰⁰ Uhr R 3.107?

Prak: im Laufe des heutigen Tages
Termine im P-Tool für jeden bereitgestellt

Rätsel

$$(X - \frac{7}{2})^2 = (Y - \frac{7}{2})^2$$

$$\Rightarrow |X - \frac{7}{2}| = |Y - \frac{7}{2}|$$

$$\Rightarrow X - \frac{7}{2} = + (Y - \frac{7}{2}) \quad \vee \quad X - \frac{7}{2} = - (Y - \frac{7}{2})$$

...

Zahlenfolgen

Bsp $n \in \mathbb{N}$

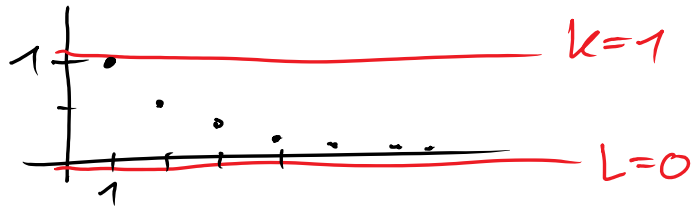
a) $a_n = \frac{1}{n} \in \mathbb{R}$

$(a_n) = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \dots$

streng monoton fallend

n.o.b. $K=1$

n.u.b. $L=0$



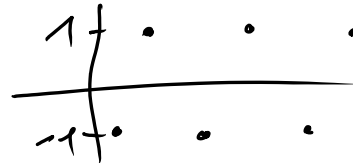
$\lim_{n \rightarrow \infty} a_n = 0$

b) $a_n = (-1)^n$ (alternierende Folge)

keine Monotonie

$(a_n) = -1, +1, -1, +1$

beschränkt, z.B. $K=1, L=-1$

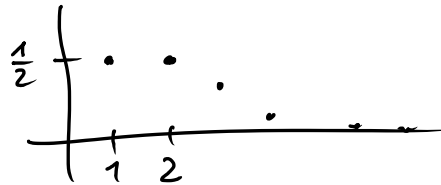


c) $a_n = \frac{n}{2^n}$

$(a_n) = \frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}, \dots$
 $\parallel$
 $\frac{1}{2}$

monoton fallend

beschränkt, z.B.: $K=1, L=0$



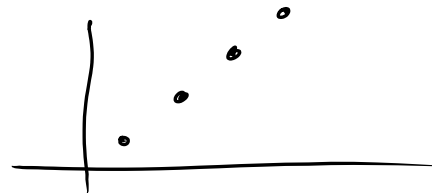
d) $a_n = n$

$(a_n) = 1, 2, 3, \dots$

streng monoton wachsend

n.u.b., z.B. $L=0$

nicht n.o.b.



$\lim_{n \rightarrow \infty} a_n = \infty$

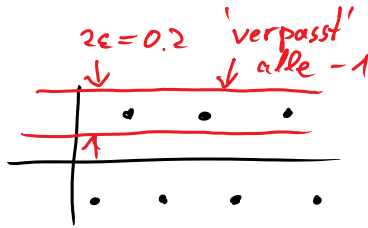
Bsp zu Grenzwert

a) $a_n = \frac{1}{n}$

$\lim_{n \rightarrow \infty} a_n = 0$ "Nullfolge"

$\frac{1}{n} \xrightarrow{n \rightarrow \infty} 0$

b) $a_n = (-1)^n$



$\lim_{n \rightarrow \infty} a_n$ ex. nicht
divergent

c) $a_n = \frac{2n-1}{3n} \Rightarrow (a_n) = \frac{1}{3}, \frac{3}{6}, \frac{5}{9}, \dots$

$$\lim_{n \rightarrow \infty} \frac{2n-1}{3n} = \lim_{n \rightarrow \infty} \frac{\cancel{n} (2 - \frac{1}{n})}{\cancel{n} \cdot 3} \stackrel{\text{Nullfolge}}{=} \frac{2-0}{3} = \frac{2}{3}$$

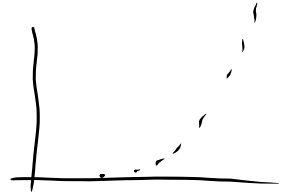
g.P.i.N.

d) $a_n = n^2 + 5 \Rightarrow (a_n) = 6, 9, 13, \dots$

divergent

$$\lim_{n \rightarrow \infty} (n^2 + 5) = +\infty \quad \text{bestimmt-divergent}$$

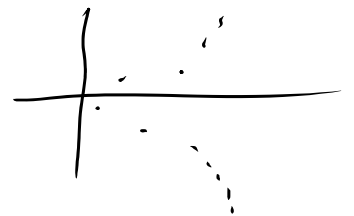
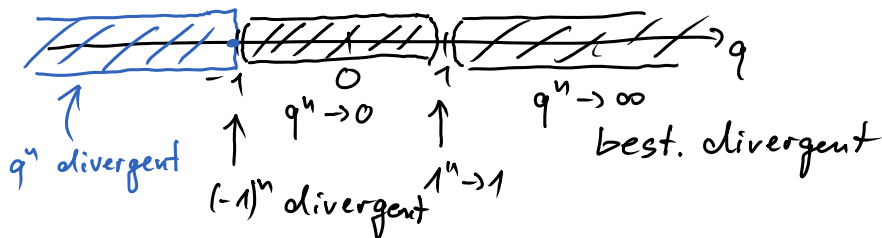
(uneig. GW ist $+\infty$)



e) $a_n = (-1)^n n^2 \Rightarrow (a_n) = -1, 4, -9, 16, \dots$

 $\lim_{n \rightarrow \infty} a_n$ ex. nicht

divergent, kein uneig. GW

f) Geometrische Folge q^n , $q \in \mathbb{R}$  q^n ist Nullfolge für $q \in]-1, 1[$

Zins 1%	Kapital K
1. Jahr	$K \cdot 1.01$
2. Jahr	$K \cdot (1.01)^2$
$\vdots$	
n . Jahr	$K \cdot \underbrace{(1.01)^n}_{q > 1}$
	$\downarrow$
	$+\infty$

Warum "0 · ∞" nicht geht "∞ - ∞"

$$1) \begin{array}{c} \frac{1}{n} \cdot n^2 = n \xrightarrow{n \rightarrow \infty} \infty \\ \downarrow \quad \downarrow \\ 0 \quad \infty \end{array}$$

$$2) \begin{array}{c} \frac{1}{n^2} \cdot n^2 = 1 \rightarrow 1 \\ \downarrow \quad \downarrow \\ 0 \quad \infty \end{array}$$

$$3) \begin{array}{c} \frac{1}{n^4} \cdot n^2 = \frac{1}{n^2} \rightarrow 0 \\ \downarrow \quad \downarrow \\ 0 \quad \infty \end{array}$$

$$a_n = n \xrightarrow{n \rightarrow \infty} \infty$$

$$b_n = 2n \rightarrow \infty$$

$$1) a_n - b_n = n - 2n = -n \xrightarrow{n \rightarrow \infty} -\infty$$

$$2) b_n - a_n = 2n - n = n \rightarrow +\infty$$

$$c_n = n + 3 \rightarrow \infty$$

$$3) a_n - c_n = n - (n + 3) = -3$$

Bsp zum Rechnen mit GW

$$a) \lim_{n \rightarrow \infty} \frac{1}{n^2} = \lim_{n \rightarrow \infty} \left(\frac{1}{n} \cdot \frac{1}{n} \right) = \left(\lim_{n \rightarrow \infty} \frac{1}{n} \right) \cdot \left(\lim_{n \rightarrow \infty} \frac{1}{n} \right) = 0 \cdot 0 = 0$$

$$b) \lim_{n \rightarrow \infty} \left(\frac{3n^3}{5n^3} \right) = \frac{\lim_{n \rightarrow \infty} 3n^3}{\lim_{n \rightarrow \infty} 5n^3} = \frac{\infty}{\infty} = \text{unentscheidbar}$$

$$\lim_{n \rightarrow \infty} \left(\frac{3 \cancel{n^3}}{5 \cancel{n^3}} \right) = \lim_{n \rightarrow \infty} \frac{3}{5} = \frac{3}{5}$$

$$c) \lim_{n \rightarrow \infty} \left(\frac{3n}{5n^4} \right) = \lim_{n \rightarrow \infty} \left(\frac{3}{5n^3} \right) = \frac{3}{\infty} = \underline{\underline{0}}$$

g.P.i.N. : größte Potenz im Nenner

$$d) \lim_{n \rightarrow \infty} \frac{-2n^2 + 4n - 5}{8n^2 - 3n + 7} = \lim_{n \rightarrow \infty} \frac{\cancel{n^2}(-2 + \frac{4}{n} - \frac{5}{n^2})}{\cancel{n^2}(8 - \frac{3}{n} + \frac{7}{n^2})} = \frac{-2}{8} = \underline{\underline{-\frac{1}{4}}}$$

↑
g.P.i.N.

$$\lim_{n \rightarrow \infty} \frac{-2n^2}{8n^3 + 7} = \lim_{n \rightarrow \infty} \frac{\cancel{n^3}(-\frac{2}{n})}{\cancel{n^3}(8 + \frac{7}{n^3})} = \frac{0}{8} = 0$$

$$\lim_{n \rightarrow \infty} \frac{-2n^3}{8n^2 + 7} = \lim_{n \rightarrow \infty} \frac{n^2(-2n)}{n^2(8 + \frac{7}{n^2})} = -\frac{1}{4}n \rightarrow \underline{\underline{-\infty}}$$

$$e) \lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) = \left(\lim_{n \rightarrow \infty} \frac{1}{n} - \lim_{n \rightarrow \infty} \frac{1}{n+1} \right) = 0 - 0 = \underline{\underline{0}}$$

$$f) \lim_{n \rightarrow \infty} \left(\frac{n^2}{n} - \frac{n^2}{n+1} \right) = (\infty - \infty) \text{ also unentscheidbar}$$

$$\left(\lim_{n \rightarrow \infty} n^2 \left(\underbrace{\frac{1}{n} - \frac{1}{n+1}}_{\rightarrow 0} \right) \right) \stackrel{!}{=} \infty \cdot 0 \quad \quad \quad " \quad \quad "$$

↙ auf Hauptnenner (H.N.) bringen

$$\lim_{n \rightarrow \infty} \left(\frac{n^2(n+1) - n^2 n}{n(n+1)} \right) = \lim_{n \rightarrow \infty} \frac{\cancel{n^2} + n^2 - \cancel{n^2}}{n^2 + n} \stackrel{!}{=} \frac{1}{1} = \underline{\underline{1}} \quad \text{g.P.i.N.}$$

Übung 2) $\lim_{n \rightarrow \infty} \left(\frac{7n^2 - 1}{3n^2 + 2} \right)^2$ 3) $\lim_{n \rightarrow \infty} \left(\frac{n^3}{(n+1)} - \frac{n^3}{(n-1)} \right)$

4) $\lim_{k \rightarrow \infty} \frac{75 \cdot 10^k + 6 \cdot 10^{2k}}{0.4 \cdot 10^{k-3} - 20 \cdot 10^{2k-2}}$

Lsg 2) $\left(\lim_{n \rightarrow \infty} \frac{7n^2 - 1}{3n^2 + 2} \right)^2 = \left(\lim_{n \rightarrow \infty} \frac{\cancel{n^2} \left(7 - \frac{1}{n} \right)}{\cancel{n^2} \left(3 + \frac{2}{n} \right)} \right)^2 = \left(\frac{7}{3} \right)^2 = \underline{\underline{\frac{49}{9}}}$

3) auf HN bringen

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{n^3(n-1) - n^3(n+1)}{(n+1)(n-1)} &= \lim_{n \rightarrow \infty} \frac{\cancel{n^4} - n^3 - (\cancel{n^4} + n^3)}{n^2 - 1} \\ &= \lim_{n \rightarrow \infty} \frac{-2n^3}{n^2 - 1} \stackrel{\text{g.P.:N}}{=} \lim_{n \rightarrow \infty} \frac{\cancel{n^2}(-2n)}{\cancel{n^2} \left(1 - \frac{1}{n^2} \right)} = \underline{\underline{-\infty}} \end{aligned}$$

4) $\lim_{k \rightarrow \infty} \frac{75 \cdot 10^k + 6 \cdot 10^{2k}}{0.4 \cdot 10^{k-3} - 20 \cdot 10^{2k} \cdot 10^{-2}} = \left(\begin{array}{l} 10^{2k} \text{ ausklammern} \\ "-2k \text{ im Exponenten} \end{array} \right) \left\{ \begin{array}{l} 10^a + b = 10^a \cdot 10^b \\ 10^a \cdot 10^{-2k} \\ 10^{a-2k} \end{array} \right.$

$$\begin{aligned} &= \lim_{k \rightarrow \infty} \frac{10^{2k} (75 \cdot 10^{k-2k} + 6 \cdot 10^{2k-2k})}{10^{2k} (0.4 \cdot 10^{k-3-2k} - 20 \cdot 0.01 \cdot 10^{2k-2k})} \\ &= \lim_{k \rightarrow \infty} \frac{75 \cdot \cancel{10^{-k}} + 6}{0.4 \cdot \cancel{10^{-k-3}} - 0.2} \\ &= \frac{6}{-0.2} = \underline{\underline{-30}} \end{aligned}$$

$\left(10^{-k} = \frac{1}{10^k} \right)$
 $= \left(\frac{1}{10} \right)^k$
 $= 9^k \text{ mit } 9=0.1$

Fixpunkt-Iteration

Mittwoch, 26. Oktober 2022 11:23

Bsp. $x + 1 = \sin(x)$ $\mid -1$ (transzendente Gl.)

$$x = \underbrace{\sin(x) - 1}_{f(x)}$$

$\downarrow$ $\downarrow$

a_{n+1} a_n

$$\arcsin(x+1) = x$$

n	a_n	a_{n+1}
1	1	$\sin(1) - 1 = a_2$
2	a_2	$\sin(a_2) - 1 = a_3$
3	a_3	...
...	...	...

Landau: $a_n = 2n^3 - n^2$ ist $O(n^3)$

weil $\frac{2n^3 - n^2}{n^3} = 2 - \frac{1}{n} \xrightarrow{n \rightarrow \infty} 2$