Aufgabe 8 Klausur 25.9.2012

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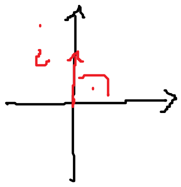
09:17

a) $z^5 = i$

Berechnung der fünften Wurzeln aus i



$i = 0 + 1i \quad \Rightarrow \quad |i| = 1$

$\varphi = 90^\circ = \frac{\pi}{2}$

Alle Beträge $\sqrt[5]{1} = 1$

$z_1 = 1\left(\cos\frac{90^\circ}{5} + i\sin\frac{90^\circ}{5}\right) = \cos 18^\circ + i\sin 18^\circ$

$= 0.951 + i \cdot 0.309$

$z_2 = \cos(18^\circ + 72^\circ) + i\sin(18^\circ + 72^\circ) = \cos 90^\circ + i\sin 90^\circ$

$= 0 + 1 \cdot i = i$

$z_3 = \cos(18^\circ + 2 \cdot 72^\circ) + i\sin(18^\circ + 2 \cdot 72^\circ) = \cos 162^\circ + i\sin 162^\circ$

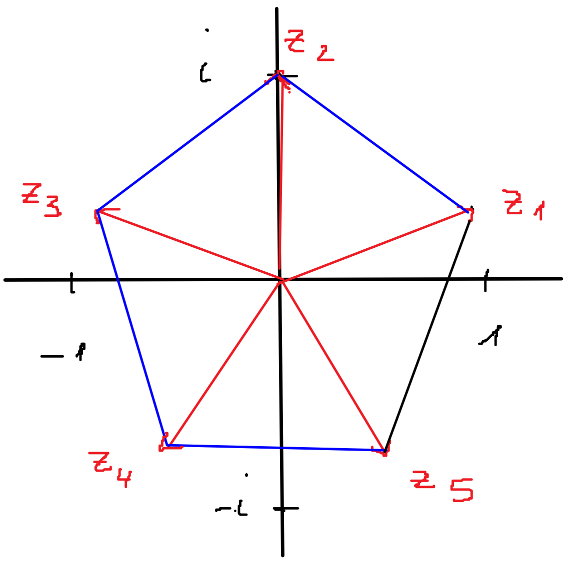
$= -0.951 + 0.309$

$z_4 = \cos(18^\circ + 3 \cdot 72^\circ) + i\sin(18^\circ + 3 \cdot 72^\circ) = \cos 234^\circ + i\sin 234^\circ$

$= -0.587 - 0.809i$

$z_5 = \cos(18^\circ + 4 \cdot 72^\circ) + i\sin(18^\circ + 4 \cdot 72^\circ) = \cos 306^\circ + i\sin 306^\circ$

$= 0.587 - 0.809i$



$\sqrt[5]{i}$

b) $y' = y^2 \cdot \sin x \qquad \frac{dy}{dx} = y^2 \cdot \sin x$

Trennung der Variablen:

$$\frac{dy}{y^2} = \sin x \, dx \quad \Big| \int$$

$$\int \frac{1}{y^2} dy = \int \sin x \, dx$$

$$-y^{-1} = -\cos x + C$$

$$\Leftrightarrow \quad -\frac{1}{y} = -\cos x + C$$

$$\Leftrightarrow \quad \frac{1}{y} = \cos x - C$$

$$y = \frac{1}{\cos x - C}$$

Anfangswert: $y(\pi) = 2$

$$\frac{1}{\cos \pi - C} = 2 \quad \Leftrightarrow \quad \frac{1}{2} = \cos \pi - C$$

$$\Leftrightarrow \quad C = \cos \pi - \frac{1}{2}$$

$$\Leftrightarrow \quad C = -1 - \frac{1}{2} = -\frac{3}{2}$$

Lösung der Anfangswertaufgabe:

$$y = \frac{1}{\cos x + \frac{3}{2}}$$