

Aufgabe 3 a)

Übergang vom Gradmaß auf Bogenmaß

$$36^\circ = \frac{360^\circ}{10} = \frac{2\pi}{10} = \frac{\pi}{5} \approx 0.628318$$

$$f(x) = \sin x$$

$$f(0) = 0$$

$$f'(x) = \cos x$$

$$f'(0) = 1$$

$$f''(x) = -\sin x$$

$$f''(0) = 0$$

$$f'''(x) = -\cos x$$

$$f'''(0) = -1$$

$$f^{(4)}(x) = \sin x$$

$$f^{(4)}(0) = 0$$

$$f^{(5)}(x) = \cos x$$

$$f^{(5)}(0) = 1$$

$$f^{(6)}(x) = -\sin x$$

Taylorpolynom 5. Grades

$$\begin{aligned} T_5(x) &= \frac{0}{0!} x^0 + \frac{1}{1!} x^1 + \frac{0}{2!} x^2 - \frac{1}{3!} x^3 + \frac{0}{4!} x^4 + \frac{1}{5!} x^5 \\ &= x - \frac{1}{6} x^3 + \frac{1}{120} x^5 \end{aligned}$$

$$T_5\left(\frac{\pi}{5}\right) = \frac{\pi}{5} - \frac{1}{6} \left(\frac{\pi}{5}\right)^3 + \frac{1}{120} \left(\frac{\pi}{5}\right)^5$$

$$= 0.628318 - \frac{1}{6} \cdot 0.24805 + \frac{1}{120} \cdot 0.09792$$

$$= 0.628318 - 0.0413416 + 0.00081605$$

$$= 0.58779245$$

Restgliedabschätzung:

$$R_5(x) \leq \left| \frac{C}{6!} \cdot x^6 \right|$$

$$|f^{(6)}(x)| \leq 1$$

$$R_5(x) \leq \left| \frac{1}{6!} \cdot 0.628318^6 \right| = 8.54417e-5 = 0.0000854417$$