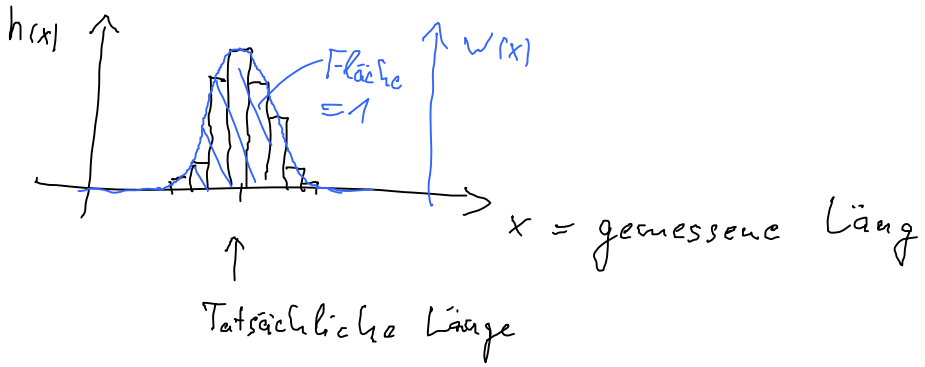
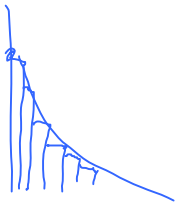
Mathe 2, 2.4.2014

Bsp Längenmessung Wahrscheinlichkeitsdichte



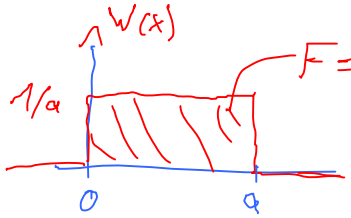
$$\int_{-\infty}^{x_0} w(x)\,dx = P(X \le x_0) = F(x_0) = \text{Verteilungsfkt an der Stelle } x_0$$



$$\int_{-\infty}^{-\infty} w(x)\,dx = 0 = \lim_{x_0 \to -\infty} F(x_0)$$

$$\int_{-\infty}^{\infty} w(x)\,dx = 1 = \lim_{x_0 \to +\infty} F(x_0)$$

Beispiel



$F = 1 = a \cdot \frac{1}{a}$ ✓

Erwartungswert $E[X] = \frac{a}{2} = \int_{-\infty}^{\infty} x\, w(x)\,dx = \int_0^a x \cdot \frac{1}{a}\,dx = \frac{1}{a} \cdot \frac{x^2}{2}\Big|_0^a$

$= \frac{1}{a}\frac{a^2}{2} = \frac{a}{2}$

Varianz $\sigma^2 = E\left[\left(X - \frac{a}{2}\right)^2\right] = \int_{-\infty}^{\infty} \left(x - \frac{a}{2}\right)^2 \cdot w(x)\, dx = \int_0^a \left(x - \frac{a}{2}\right)^2 \frac{1}{a}\, dx$

$= \frac{1}{a} \int_{-\frac{a}{2}}^{+\frac{a}{2}} z^2\, dz = \frac{1}{a} \frac{1}{3} z^3 \Big|_{-a/2}^{a/2} = \frac{2}{a} \cdot \frac{1}{3} z^3 \Big|_0^{a/2}$ $\quad \left| z = x - \frac{a}{2} \right.$

$= \frac{2}{3} \frac{1}{a} \left(\frac{a}{2}\right)^3 = \frac{1}{12} a^2$