MA 2 V 30.04.2014

Extra V: Mi, 7.5, $13^{00} - 14^{30}$, 0.401

keine V Mi, 14.5.

V Mo 12.5., 11:00

keine Ü Mo, 12.5. 9^{00}

Ü Mo, 12.5. 13^{00}

Komplexe Zahlen

$z = a + ib$;

$$z \cdot z^* = (a + ib)(a - ib)$$

$$= a^2 - i^2 b^2 + \cancel{iba - iba}$$

$$= a^2 + b^2$$

$$= |z|^2 \quad (\text{Betrag von } z)^2$$

$|z| = \sqrt{z \cdot z^*}$

$\boxed{i^2 = -1}$

Ü a) $i(3 - 2i) = 3i - 2i^2 = 3i + 2$ $\quad$ Re: 2, Im: 3

b) $|3 - 4i| = \sqrt{(3-4i)(3+4i)} \underset{\substack{\uparrow \\ \text{3. bin. Formel}}}{=} \sqrt{3^2 - (4i)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = \underline{\underline{5}}$

c) $\dfrac{3+4i}{2-i} = \dfrac{(3+4i)(2+i)}{(2-i)(2+i)} = \dfrac{6 + 4i^2 + 3i + 8i}{2^2 - i^2} = \dfrac{6 - 4 + i(3+8)}{2^2 + 1^2}$

$$= \frac{2 + 11i}{5} = \frac{2}{5} + i\,\frac{11}{5} \qquad \text{Re: } \frac{2}{5}, \ \text{Im: } \frac{11}{5}$$

Eulersche Formel $\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi}$

Folgerung $e^{-i\varphi} = e^{i(-\varphi)} = \cos(-\varphi) + i\sin(-\varphi)$

$\Leftrightarrow \boxed{e^{-i\varphi} = \cos(\varphi) - i\sin(\varphi)}$

Taylor für $e^{i\varphi}$:

$$e^{i\varphi} = 1 + \frac{(i\varphi)}{1!} + \frac{(i\varphi)^2}{2!} + \frac{(i\varphi)^3}{3!} + \frac{(i\varphi)^4}{4!} + \frac{(i\varphi)^5}{5!} + \dots$$

$i^3 = i\,i^2 = -i$

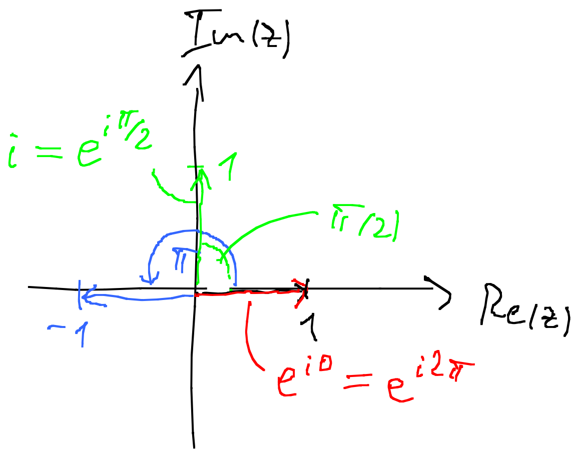
$i^4 = i^2 i^2 = 1$

$$= 1 + \frac{i\varphi}{1!} - \frac{\varphi^2}{2!} - \frac{i\varphi^3}{3!} + \frac{\varphi^4}{4!} + \frac{i\varphi^5}{5!} + \dots$$

$$= \underbrace{\left(1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} \mp \dots\right)}_{\cos(\varphi)} + i\underbrace{\left(\varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} + \dots\right)}_{\sin(\varphi)}$$

Ü

z	φ	$Re(z)$ $\cos(\varphi)$	$Im(z)$ $\sin(\varphi)$
e^{i0}	0	1	0
$e^{i\pi}$	π	-1	0
$e^{i2\pi}$	2π	1	0
$e^{i\pi/2}$	$\pi/2$	0	1

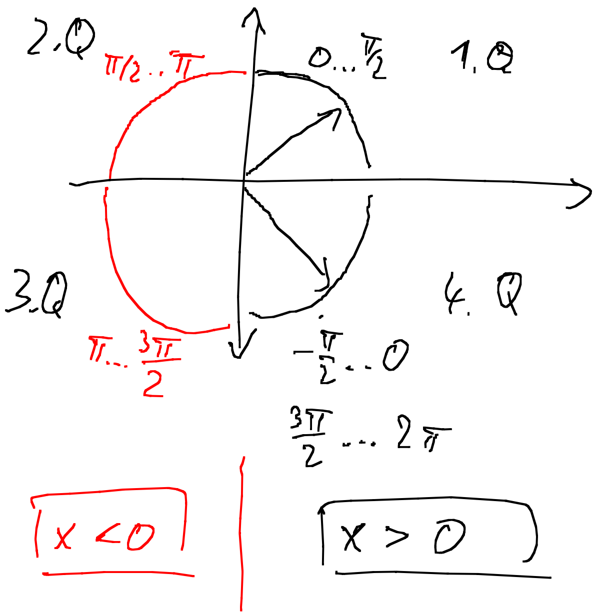


$$1 = e^{i0} = e^{i2\pi} = e^{i4\pi} = \dots e^{i2n\pi}, \quad n \in \mathbb{Z}$$

$$i = e^{i\pi/2}$$

Umrechnung "kartesisch $\to$ polar"

$\operatorname{atan}\left(\frac{y}{x}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



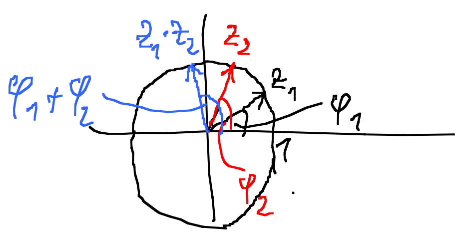
Beispiel

$z = -2 + 2\sqrt{3}\,i$ Umrechnung Polar:

$r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \underline{\underline{4}}$ $\quad \operatorname{Re}(z) = -2 < 0 \Rightarrow 2.Q$

$\varphi = \operatorname{atan}\left(\frac{2\sqrt{3}}{-2}\right) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3} \mathrel{\hat{=}} 120°$

Multiplikation ist Drehung oder Drehstreckung



Beispiel zu Potenz mit c = rationale Zahl

$$\boxed{i = e^{i\pi/2}}$$

$$z = i^{5/3} \quad ?$$

$$= \left(e^{i\frac{\pi}{2}}\right)^{\frac{5}{3}} = \left(e^{i\frac{5\pi}{2}}\right)^{\frac{1}{3}}$$

$$= \left(e^{i\left(\frac{5\pi}{2} + 2k\pi\right)}\right)^{\frac{1}{3}}$$

$$= e^{i\left(\frac{5\pi}{6} + \frac{2k\pi}{3}\right)} \quad , \; k = 0, 1, 2$$

k	$\frac{2k\pi}{3}$
0	0
1	$\frac{2\pi}{3}$
2	$\frac{4\pi}{3}$
~~3~~	~~$\frac{6\pi}{3} = 2\pi = 0$~~

Drei Lösungen (allg: $k = 0, \ldots, q-1$)

$$z_1 = e^{i\frac{5\pi}{6}}$$

$$z_2 = e^{i\left(\frac{5\pi}{6} + \frac{2\pi}{3}\right)} = e^{i\frac{9\pi}{6}} = e^{i\frac{3\pi}{2}} = -i$$

$$z_3 = e^{i\frac{13\pi}{6}} = e^{i\frac{\pi}{6}}$$