Mathe V 7.05.2014

Meinprof.de

Quadratische Ergänzung

2. binom Formel

$z^2 - 2bz + b^2 = (z-b)^2$

$z^2 - 2bz + c = 0$

$z^2 - 2bz + b^2 - b^2 + c = 0$

$z^2 - 2bz + b^2 = b^2 - c$

$(z - b)^2 = b^2 - c$

$z - b = \pm\sqrt{b^2 - c}$

$z = b \pm \sqrt{b^2 - c}$

Ü

$z^2 - (4-2i)z + (5-4i) = 0$

$z^2 - 2\underbrace{(2-i)}_{b}z + \underbrace{(2-i)^2}_{b^2} - (2-i)^2 + (5-4i) = 0$

$(z - (2-i))^2 - (2-i)^2 + (5-4i) = 0$

$\Leftrightarrow (z - (2-i))^2 = (2-i)^2 - (5-4i)$

$\Leftrightarrow (z - (2-i))^2 = 4 - \cancel{4}i + i^2 - 5 + \cancel{4}i$

$\Leftrightarrow (z - (2-i))^2 = -1 - 1 = -2$

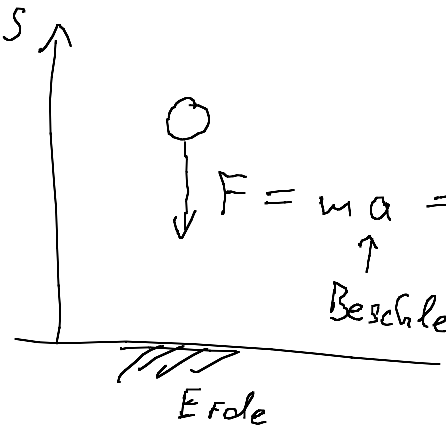
$z = (2-i) \pm \sqrt{-2}$

$= 2 - i \pm \sqrt{2}\, i$

$= 2 + (-1 \pm \sqrt{2})i$

Differentialgleichungen

„freier Fall"



$F = m\,a = m(-g)$

$9.81\ m/s^2$

↑ Beschleunigung

$a = -g = const$

$a(t)$ = Beschleunigung
= Änderung der Geschw.

$a(t) = v'(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

$v(t) = s'(t)$ = Änderung der Strecke

Zusammen $\boxed{a(t) = s''(t)}$

$\boxed{s''(t) = -g}$ einfache DGL

Lineare Algebra

$A\vec{x} = \vec{b}$
inhomogen

$A\vec{x} = 0$
homogen

Ü a) $y'(x) = -2x$ 1. Ordnung, explizit, linear, inhomogen

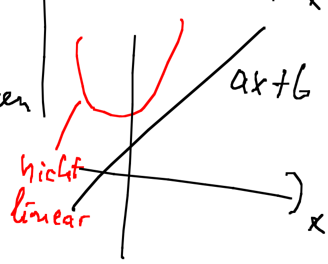
b) $x + yy' = 0$ 1. Ordnung, implizit, nichtlinear,

c) $s''(t) = g$ 2. Ordnung, explizit, linear, inhomogen

d) $y'' + y = 0$ 2. Ordnung, explizit, linear, homogen

e) $2y'' - 4y' + 20y = \cos(\omega x)$ 2. Ordn., explizit, linear mit konst. Koeff., inhomogen

linear in x



Lösung v. DGL

1) Direkte Integration

$$s''(t) = -g \qquad | \int$$

$$\int s''(t)\,dt = \int(-g)\,dt$$

$$s'(t) = -gt + C \qquad | \int$$

$$\int s'(t)\,dt = \int(-gt + C)\,dt$$

$$\underline{\underline{s(t) = -\tfrac{1}{2}gt^2 + Ct + D}}$$

Auf. geschw.

$s(0) = D$ = Anfangsort

$s'(0) = C$ = Anfangsgeschw

Hinweis

$$\int t^\alpha\,dt = \frac{1}{\alpha+1}\,t^{\alpha+1}$$

Ü direkte Integration

$$s''(t) = \frac{1}{2\sqrt{t}} = \frac{1}{2}\,t^{-\frac{1}{2}} \qquad | \int$$

$$\int s''(t)\,dt = \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$s'(t)\,dt = \cancel{\tfrac{1}{2}}\,\frac{1}{\cancel{(\frac{1}{2})}}\,t^{\frac{1}{2}} + C_1 \qquad | \int$$

$$\int s'(t)\,dt = \int\left(t^{\frac{1}{2}} + C_1\right)dt$$

$$\underline{\underline{s(t) = \tfrac{2}{3}\,t^{\frac{3}{2}} + C_1 t + C_2}}$$

allgemeine Lsg.

spezielle Lsg für

$s'(1) = 2 = 1^{\frac{1}{2}} + C_1$

$\Leftrightarrow \quad C_1 = 1$

$s(1) = 2$

$\frac{2}{3}\,1^{\frac{3}{2}} + 1 \cdot 1 + C_2 = 2$

$\Leftrightarrow \quad \frac{5}{3} + C_2 = \frac{6}{3}$

$\Leftrightarrow \quad C_2 = \frac{1}{3}$

$$\underline{\underline{s_p(t) = \tfrac{2}{3}\,t^{\frac{3}{2}} + t + \tfrac{1}{3}}}$$

partikuläre Lsg

2) Trennung der Veränderlichen (y, x)

$$y'(x) = y$$
$$\frac{dy}{dx} = y \quad | :y, \cdot dx$$
$$\frac{dy}{y} = dx \quad | \int$$
$$\int \frac{dy}{y} = \int dx$$
$$\ln(y) = x + C \quad | e^{()}$$
$$\boxed{y = e^{x+C}}$$

Probe $y'(x) = e^{x+C} \cdot 1 = y(x)$

Ü a) $y'(x) = x^2\sqrt{y}$

b) $y'(x) = y \cdot \sin(x)$

Ü a) $y'(x) = x^2\sqrt{y}$

$$\frac{dy}{dx} = x^2\sqrt{y} \quad | \cdot dx, :\sqrt{y}$$
$$\frac{dy}{\sqrt{y}} = x^2\, dx \quad | \int$$
$$\int y^{-\frac{1}{2}} dy = \int x^2 dx$$
$$2y^{+\frac{1}{2}} = \int x^2 dx$$
$$2y^{\frac{1}{2}} = \frac{1}{3}x^3 + C$$

$$\Rightarrow \quad y = \left(\frac{1}{6}x^3 + \frac{C}{2}\right)^2$$

Probe: $y'(x) = 2\left(\frac{1}{6}x^3 + \frac{C}{2}\right) \cdot \frac{3}{6}x^2$

Kettenregel

$$\Rightarrow y'(x) = \sqrt{y} \cdot x^2$$

$\boxed{\checkmark}$

b) $y' = y \sin(x)$

$$\int \frac{dy}{y} = \int \sin(x)\, dx \iff \ln(y) = -\cos(x) + C \quad | \, e^{\cdot}$$

$$\iff y = e^{-\cos(x) + C} = \underline{\underline{D e^{-\cos(x)}}} \qquad (D = e^{C})$$

Probe $y'(x) = D e^{-\cos(x)} \cdot \sin(x) = y(x) \cdot \sin(x)$ $\boxed{\checkmark}$