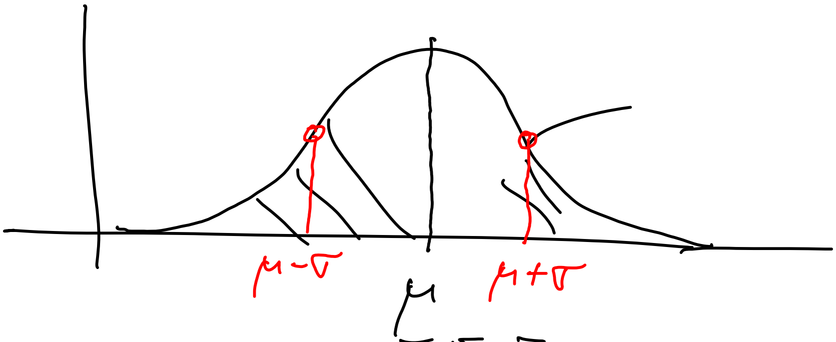
V MA2 – 11.5.2015



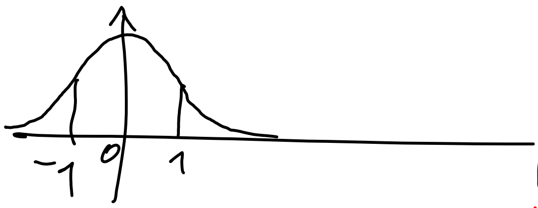
Normalverteilung $N(\mu;\sigma)$

$\mu = E[X] =$ Mittelwert

$\sigma =$ Standardabweichung

Standardnormalverteilung: $\mu = 0,\ \sigma = 1$

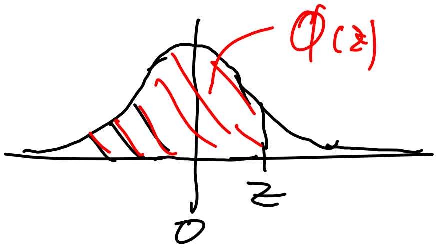
$N(0;1)$



Für Stand.normalverteilung

$$\boxed{\Phi(z) = P(X \leq z)}$$

für $X \in N(0;1)$



Zu den Regeln in Satz 10-14

Nr. 5: Regel 1 verwenden und "hinten" anfangen:

Nr. 1: $1-q = \Phi(-z_q)$

$\iff 1-q = 1-\Phi(z_q)$

$\iff q = \Phi(z_q)$

Nr. 6: $z_q = \dfrac{x_q-\mu}{\sigma}$

$\iff z_q\sigma = x_q-\mu$

$\iff z_q\sigma+\mu = x_q$

Beispiel Normalverteilungs-Aufgaben

$X \in N(\mu = 4, \sigma = 2)$

Wie groß ist a) $P(X \leq 6)$, b) $P(X \geq 2)$

c) $P(3.8 < X \leq 7)$

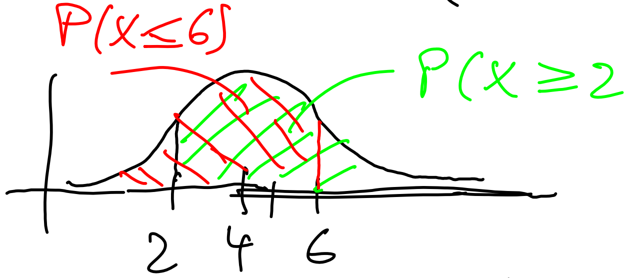
Lsg a) $P(X \leq 6) = \Phi\left(\frac{6-4}{2}\right) = \Phi(1) \overset{\text{Tab.}}{=} 84.1\%$

b) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X \leq 2) =$

$= 1 - \Phi\left(\frac{2-4}{2}\right) = 1 - \Phi(-1)$

$\overset{Nr.1}{=} 1 - (1 - \Phi(1)) = \Phi(1) = 84.1\%$



c) $P(3.8 < X \leq 7) \overset{Nr.4}{=} P(X \leq 7) - P(X \leq 3.8)$

$= \Phi\left(\frac{7-4}{2}\right) - \Phi\left(\frac{3.8-4}{2}\right) = \Phi(1.5) - \Phi(-0.1)$

$\overset{Nr.1}{=} \Phi(1.5) - (1 - \Phi(0.1)) \overset{\text{Tab.}}{=} \underline{47.3\%}$

Beispiel 1

"zum unteren 0.06-Quantil" $\hat{=}$ "die 6% Kleinsten"

Wahrsch. $q = 6\%$ gegeben, $\mu = 1.75$, $\sigma = 0.2$

Gesucht ist b

Aufgabentyp 2: $6\% = \Phi(z_q) = 0.06$

$\Leftrightarrow \Phi(-z_q) = 0.94$

$\overset{\text{Tab}}{\Rightarrow} -z_q = 1.56 \Leftrightarrow z_q = -1.56$

Nr. 6.

$$\Rightarrow b = x_q = \sigma z_q + \mu = 0.2\cdot(-1.56) + 1.75 = \underline{\underline{1.438}}$$

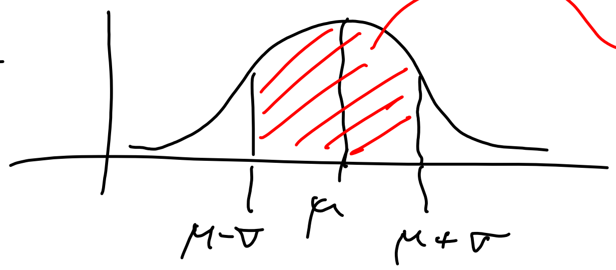
Ü1 $X \in N(1.75,\ 0.2)$

$$P(X > 2.0) = 1 - P(X \le 2.0)$$

$$= 1 - \Phi\left(\frac{2 - 1.75}{0.2}\right) = 1 - \Phi(1.25)$$

$$= 1 - 0.8944 = \underline{\underline{10.56\,\%}}$$

Ü2



$$P(\mu - \sigma \le X \le \mu + \sigma)$$

$$= P(X \le \mu + \sigma) - P(X \le \mu - \sigma)$$

$$= \Phi\left(\frac{\mu + \sigma - \mu}{\sigma}\right) - \Phi\left(\frac{\cancel{\mu} - \cancel{\sigma} - \mu}{\sigma}\right)$$

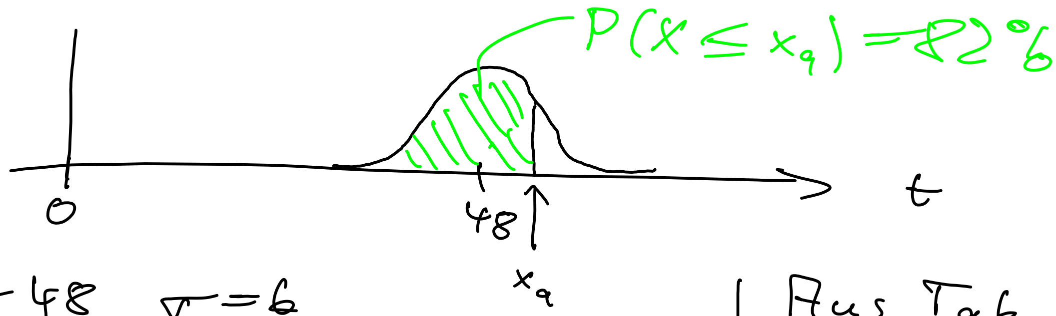
$$= \Phi\left(\frac{+\sigma}{\sigma}\right) - \Phi\left(\frac{-\sigma}{\sigma}\right) = \Phi(1) - \Phi(-1)$$

$$= \Phi(1) - (1 - \Phi(1)) = 2\Phi(1) - 1 = \underline{\underline{68.2\%}}$$

Analog: $P(\mu - 2\sigma \le X \le \mu + 2\sigma) = 2\Phi(2) - 1 = \underline{\underline{95.5\%}}$

$P(\mu - 3\sigma \le X \le \mu + 3\sigma) = 2\Phi(3) - 1 = \underline{\underline{99.7\%}}$

Ü3



$\mu = 48,\ \sigma = 6$

$$0.82 = P(X \le x_q)$$

$$\Leftrightarrow 0.82 = \Phi\left(\frac{x_q - \mu}{\sigma}\right) = \Phi(z_q)$$

Aus Tab

$$z_q = 0.92 \Leftrightarrow$$

$$\frac{x_q - \mu}{\sigma} = 0.92 \Leftrightarrow$$

$$x_q = \sigma \cdot 0.92 + \mu$$

$$= \underline{53.52}\ h$$

$x_1 = 53,52\,h$