V MA2 – 10.5.2017

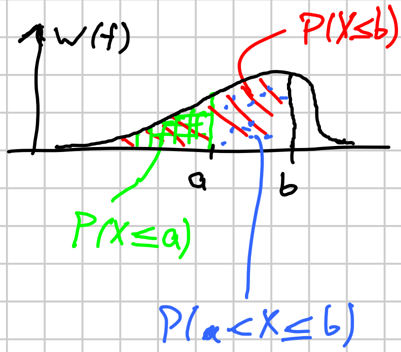
Mo 15.5 Ü 9 Uhr 1.400

V 11 Uhr "

keine 13 Uhr Übung

keine V Mittwoch 17.5

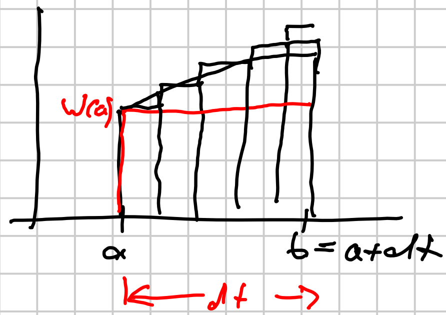
Wahrsch. dichte, X stetige Zuf. Var.



$$P(a < X \le b) = P(X \le b) - P(X \le a)$$
$$= \int_{-\infty}^{b} w(t)\,dt - \int_{-\infty}^{a} w(t)\,dt$$
$$= \int_{a}^{b} w(t)\,dt$$
$$= F(b) - F(a)$$

$P(a < X \le a+dt) \approx w(a)\,dt$

für kleines dt



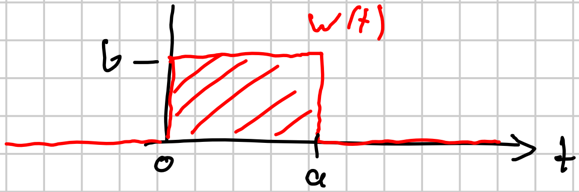
Erwartungswert

$$E(X) = \int_{-\infty}^{\infty} t\, w(t)\,dt \approx \sum_{n=-\infty}^{+\infty} t_n \underbrace{w(t_n)\,\Delta t}_{p(t_n)} \qquad \Delta t = t_{n+1} - t_n$$
$$\sum_{n=-\infty}^{\infty} t_n\, p(t_n)$$

Linearität Erwartungswert

$$E(aX+b) = \int_{-\infty}^{\infty} (at+b)\, w(t)\,dt$$
$$= \int_{-\infty}^{\infty} at\, w(t)\,dt + \int_{-\infty}^{\infty} b\, w(t)\,dt$$
$$= a \underbrace{\int_{-\infty}^{\infty} t\, w(t)\,dt}_{E(X)} + b \underbrace{\int_{-\infty}^{\infty} w(t)\,dt}_{=1}$$
$$= a\,E(X) + b$$

Beispiel Gleichverteilung $X \in [0, a]$

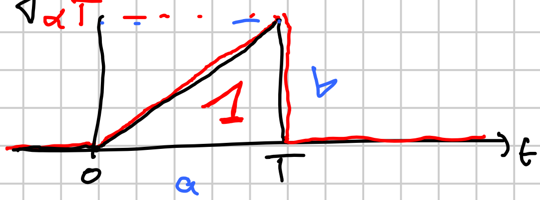


Rechteck $b \cdot a = 1$

$\Rightarrow b = \frac{1}{a}$

$$w(t) = \begin{cases} \frac{1}{a} & \text{für } t \in [0, a] \\ 0 & \text{sonst} \end{cases}$$

Übung



$$w(t) = \begin{cases} \alpha T & \text{für } t \in [0, T] \\ 0 & \text{sonst} \end{cases}$$

Für jede Wahrscheinlichkeitsdichte $w(t)$ gilt

$$\int_{-\infty}^{\infty} w(t)\, dt = 1$$

a) Fläche Dreieck $\frac{ab}{2}$. Hier $a = T$, $b = \alpha T$

$$\Rightarrow 1 = \frac{T \cdot \alpha T}{2} \quad \Leftrightarrow \quad \underline{\underline{\alpha = \frac{2}{T^2}}}$$

Alternativ: Integral rechnen

$$1 = \int_0^T w(t)\, dt = \int_0^T \alpha t\, dt = \alpha \frac{1}{2} t^2 \Big|_0^T = \alpha \frac{T^2}{2} \Rightarrow \underline{\underline{\alpha = \frac{2}{T^2}}}$$

b) $$\mu = E(X) = \int_0^T t\, \alpha t\, dt = \frac{2}{T^2} \int_0^T t^2\, dt = \frac{2}{\cancel{T^2}} \cdot \frac{1}{3} T^{\cancel{3}}$$

$$= \frac{2}{3} T = 0.\overline{6}\, T$$

c) Gesucht ist t_m mit $P(X \leq t_m) = 50\%$

$$\ldots \qquad t_m = \frac{T}{\sqrt{2}} = \underline{\underline{0.707\, T}}$$