[V 22.5.2017 MA2]

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$ Bayes

$$P(B) = \sum_{i=1}^{n} P(B|A_i)\,P(A_i)$$ $A_1 \cup A_2 \cup \dots A_n = \Omega$

Übung

	A_1	A_2
$P(A_i)$	0.4	0.6
$P(B\|A_i)$	0.1	0.2

Was ist $P(A_i|B)$, $i=1,2$?

" " $P(B)$?

Lsg

$$P(B) = \underbrace{0.1 \cdot 0.4}_{} + \underbrace{0.2 \cdot 0.6}_{} = 0.16 = 16\%$$

$$P(A_1|B) = \frac{P(B|A_1)\,P(A_1)}{P(B)} = \frac{0.1 \cdot 0.4}{0.16} = 25\%$$

$$P(A_2|B) = \frac{P(B|A_2)\,P(A_2)}{P(B)} = \frac{0.2 \cdot 0.6}{0.16} = 75\%$$

Σ 100%

Zu stat. Unabh.

$$P(A \cap B) \underset{S10\text{-}6}{=} P(B|A) \cdot P(A) \Rightarrow$$

$$P(A \cap B) \overset{\uparrow}{=} P(B) \cdot P(A)$$

stat. unabh. D10-13

$\Rightarrow$ Falls A, B stat. unabh. u. $P(A) \neq 0$

$$P(B|A) = P(B)$$

Ziegen-Problem

Lösung

A_1: Auto hinter Tür 1
A_2: " " " 2
A_3: " " 3

Kandidat wählt zuerst Tür 3

M_1: Moderator öffnet Tür 1
→ Ziege!

$P(A_2 | M_1) = ?$ Ziel

$P(A_3 | M_1) = ?$

(*) $P(M_1 | A_1)$, $P(M_1 | A_2)$, $P(M_1 | A_3) = ?$

$P(M_1) = ?$

$P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$

Moderator öffnet <u>nie</u> Tür mit Auto
" " Tür Kandidat

Kand. T3 ⇒ M entweder T1 oder T2

$P(M_1 | A_1) = 0$
$P(M_1 | A_2) = 100\% = 1$
$P(M_1 | A_3) = 50\% = \frac{1}{2}$

$P(M_1) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{3}{6} = \frac{1}{2} = 50\%$

$P(A_2 | M_1) = \frac{P(M_1 | A_2)\, P(A_2)}{P(M_1)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} = \underline{66\%}$

$P(A_3 | M_1) = \frac{P(M_1 | A_3)\, P(A_3)}{P(M_1)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} = \underline{33\%}$

ii Binomial / Hypergeom. Vert

Aufg

Urne $N = 60$, 6 weiß (w), 2x ziehen

Wie groß ist $P(\text{"w-w"})$?

Lsg

a) z m z → Binomialverteilung

~~$\binom{60}{2}$~~

n = Länge Bernoulli Kette

$= 2$

$p = \frac{6}{60} = \frac{1}{10}$ $\qquad k = 2$

$$P(\text{"w-w"}) = P(X=2) = \binom{2}{2} \cdot p^2 \cdot (1-p)^0$$

$$= 1 \cdot (0.1)^2 = 0.01 = \underline{1\,\%}$$

b) z o z → Hypergeometr. Vert. , $k = 2$ weiße

$$P(\text{"w-w"}) = \frac{\binom{6}{2}\binom{54}{0}}{\binom{60}{2}} = \frac{\binom{6}{2}}{\binom{60}{2}} = \underline{0.847\,\%}$$