V MA 2 – 24.05.2017

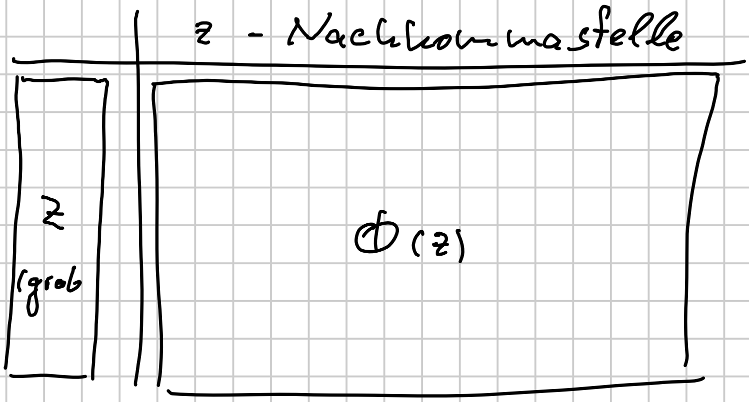
Standardnormalverteilung $N(0,1)$

$$w(t) = \varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

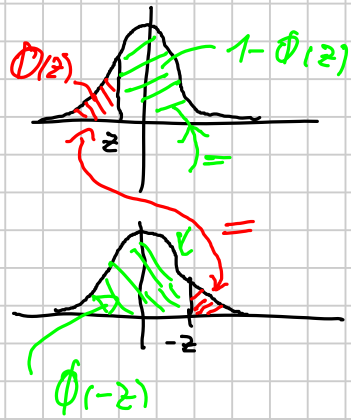
Allgemeine Normalverteilung $N(\mu, \sigma)$

$$w(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

Tabelle für $\Phi(z)$



$\Phi(z)$ für $z < 0$??



$$\Phi(-z) = 1 - \Phi(z)$$

$$\Phi(z) = 1 - \Phi(-z)$$

Bsp. $\Phi(-0.3) = 1 - \Phi(0.3)$

<u>Bsp Körpergröße</u>

$\mu = 1.75$

$\sigma = 0.20$

Gesucht: b so daß $P(X \leq b) = 0.06 = 6\%$

D.h. Suche $z_{6\%} = z_{0.06}$ der Standardnormalvert.

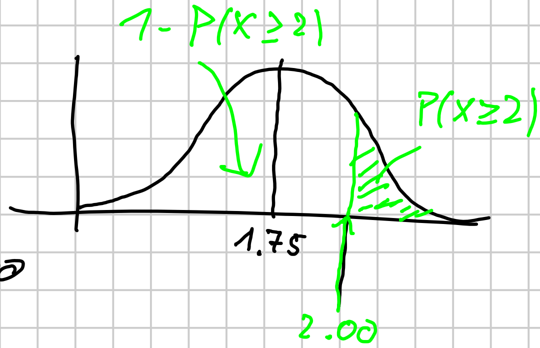
$$0.06 = P(Z \leq z_{0.06}) = \Phi(z_{0.06})$$

$$\Leftrightarrow \quad 0.94 = 1 - 0.06 = \Phi(-z_{0.06})$$

aus Tabelle: $-z_{0.06} = 1.56 \quad \Rightarrow \quad z_{0.06} = -1.56$

Regel 6 $\Rightarrow$ $x_q = x_{0.06} = \sigma z_{0.06} + \mu = 0.2\,(-1.56) + 1.75 = \underline{\underline{1.438}}$

$\boxed{Ü1,\ Ü2,\ Ü4}$



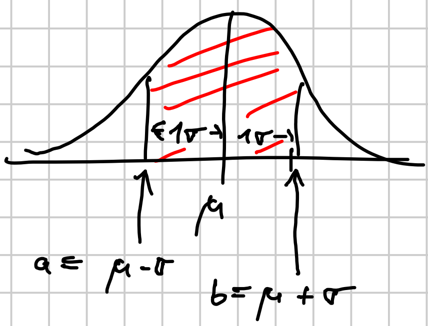
Lsg zu Ü1

$\mu = 1.75$
$\sigma = 0.20$

Gesucht ist $P(X \geq b)$ mit $b = 2.00$

$$
\begin{aligned}
P(X \geq 2.0) &= 1 - P(X < 2.0) \\
&= 1 - \Phi\left(\frac{2.0-\mu}{\sigma}\right) = 1 - \Phi\left(\frac{2.0 - 1.75}{0.2}\right) \\
&= 1 - \Phi(1.25) \underset{\text{Tab.}}{=} 1 - 0.8944 \\
&= \underline{\underline{0.1056}} \approx 10\%
\end{aligned}
$$

Lsg zu Ü2:



$$
\begin{aligned}
&P(a \leq X \leq b) \\
&= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right) \\
&= \Phi\left(\frac{\cancel{\mu}+\cancel{\sigma}-\cancel{\mu}}{\cancel{\sigma}}\right) - \Phi\left(\frac{\cancel{\mu}-\cancel{\sigma}-\cancel{\mu}}{\cancel{\sigma}}\right) \\
&= \Phi(1) - \Phi(-1) \\
&= \Phi(1) - (1 - \Phi(1)) \\
&= 2\Phi(1) - 1 \\
&= \underline{\underline{68.2\%}}
\end{aligned}
$$

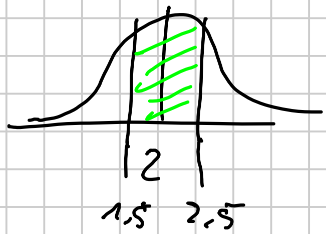
Analog:

$2\sigma \rightarrow 2\Phi(2) - 1 = 95.5\%$

$3\sigma \rightarrow 2\Phi(3) - 1 = 99.7\%$

Lsg zu Ü4:

$\mu = 2$
$\sigma = 1$

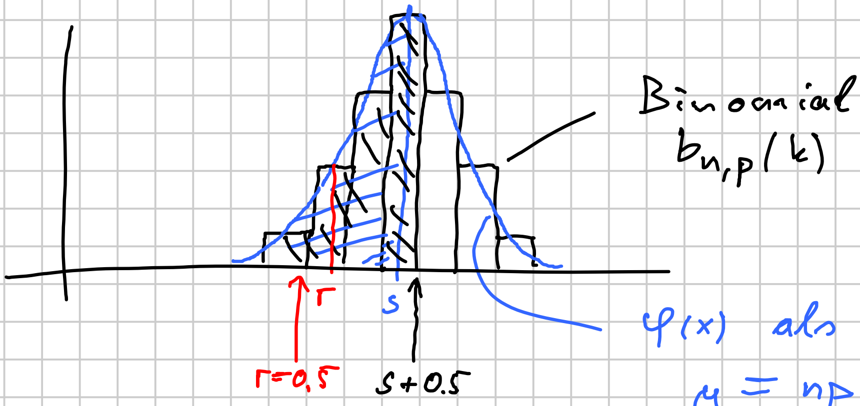


$$
\begin{aligned}
P(1.5 \leq X \leq 2.5) &= \Phi\left(\frac{2.5-2}{1}\right) - \Phi\left(\frac{1.5-2}{1}\right) \\
&= \Phi(0.5) - \Phi(-0.5) \\
&= \Phi(0.5) - (1 - \Phi(0.5))
\end{aligned}
$$

$= 2\Phi(0.5) - 1$

$= 2 \cdot 0.6915 - 1 \quad = \underline{38.29\,\%}$

Binomial vert. vs. Gauss



$\varphi(x)$ als $N(\mu, \sigma)$

$\mu = np$

$\sigma = \sqrt{np(1-p)}$