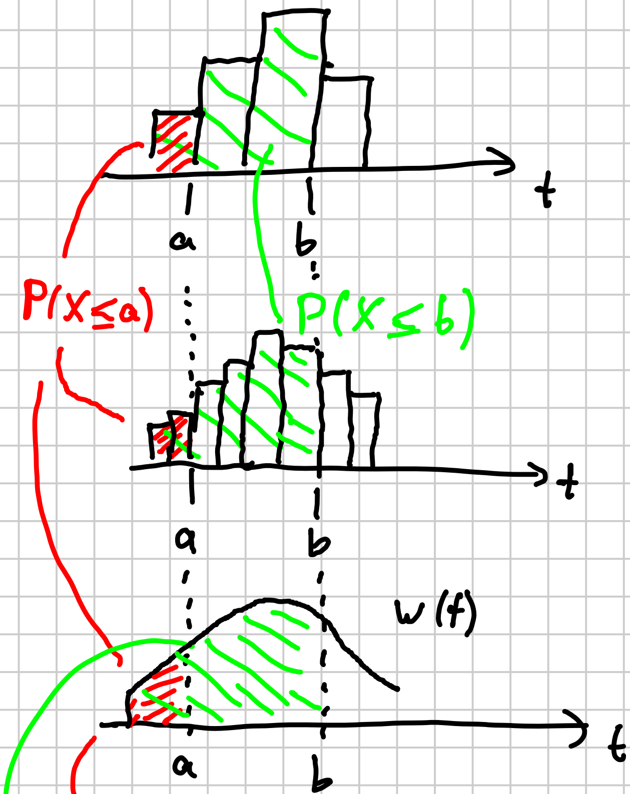
Anschauliche Herleitung Wahrsch. dichte



$$P(X \le a) + P(a < X \le b) = P(X \le b)$$

$$\Rightarrow \quad P(a < X \le b) = P(X \le b) - P(X \le a)$$

$$= F(b) - F(a)$$

$$P(X \le a) = \int_{-\infty}^{a} w(t)\,dt$$

$$P(X \le b) = \int_{-\infty}^{b} w(t)\,dt$$

$$P(a < X \le b) = \int_a^b w(t)\,dt$$

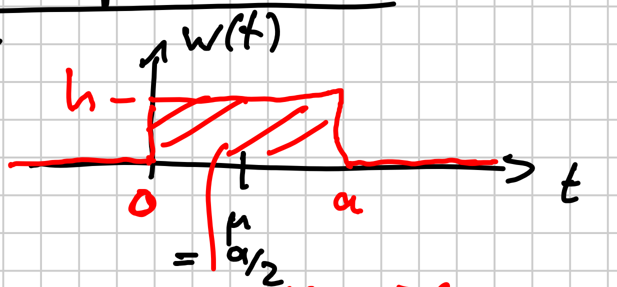
$$= \int_{-\infty}^{b} w(t)\,dt - \int_{-\infty}^{a} w(t)\,dt$$

$$= F(b) - F(a)$$

$$\boxed{F(t): \text{ Stammfkt zu } w(t)}$$

Beispiel Varianz für stetige Zuf. variable

Sei X gleichverteilt in $[0, a]$



$$w(t) = \begin{cases} \frac{1}{a} & f \quad 0 \le t \le a \\ 0 & \text{sonst} \end{cases}$$

100% = Fläche 1

$\Rightarrow \quad h = \frac{1}{a}$

(denn $a \cdot \frac{1}{a} = 1$)

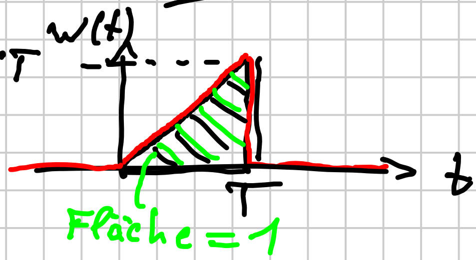
Was ist $E(X)$ und $Var(X)$?

$$\mu = E(X) = \int_{-\infty}^{\infty} t \; w(t)\,dt$$

$$= \int_0^a t \cdot \frac{1}{a}\,dt$$

$$= \frac{1}{a} \cdot \frac{1}{2} t^2 \Big|_0^a = \underline{\underline{\frac{a}{2}}}$$

$$\sigma^2 = Var(X) = \int_0^a (t-\mu)^2 \; w(t)\, dt$$
$$= \int_0^a \left(t-\frac{a}{2}\right)^2 \cdot \frac{1}{a} \; dt$$
$$= \frac{1}{a}\left[\frac{1}{3}\left(t-\frac{a}{2}\right)^3\right]_0^a$$
$$= \frac{1}{a}\left[\frac{1}{3}\left(\frac{a}{2}\right)^3 - \frac{1}{3}\left(-\frac{a}{2}\right)^3\right] = \underline{\underline{\frac{a^2}{12}}}$$



Ü4

$$w(t) = \begin{cases} \alpha t & 0 \le t \le T \\ 0 & \text{sonst} \end{cases}$$

nur $at + b$) Was ist $E(X)$?

$$E(X) = \int_0^T t\, w(t) = \int_0^T t\, \alpha t\, dt = \alpha \int_0^T t^2 dt = \alpha \frac{1}{3} t^3 \Big|_0^T = \frac{\alpha T^3}{3}$$

Was ist α?

Fläche unter $w(t)$ muss 1 sein

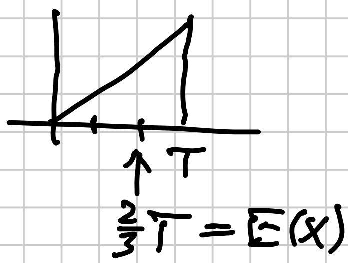
Lsg 1: Dreiecksfläche $\frac{T \cdot \alpha T}{2} = 1 \Rightarrow \alpha = \frac{2}{T^2}$

Lsg 2: $\int_0^T w(t)\, dt = \int_0^T \alpha t\, dt = 1 \Rightarrow \alpha \frac{1}{2} T^2 = 1$

$$\Rightarrow \alpha = \frac{2}{T^2}$$

Damit wird

$$E(X) = \frac{\alpha T^3}{3} = \frac{2}{T^2} \cdot \frac{T^3}{3} = \underline{\underline{\frac{2}{3} T}}$$



Bei Gleichverteilung wäre E(X)=(1/2)T.
Bei Dreiecksverteilung ist E(X)=(2/3)T höher, was auch anschaulich klar ist, da höhere X-Werte eine höhere Wahrscheinlichkeit haben.