Aufgabe 5

Klausur 26.09.2011

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$$W = \frac{19}{32} + \frac{1}{2}\alpha^2 + \frac{3}{4}(T-2) - \alpha(T-2) - \frac{1}{3}\alpha^3 - \frac{1}{2}(T-2)^2$$

$$W_\alpha = \alpha - (T-2) - \alpha^2 = 0 \quad (1)$$

$$W_T = \frac{3}{4} - \alpha - (T-2) = 0 \quad (2)$$

$$\Rightarrow \frac{3}{4} - \alpha = T-2 \quad (3)$$

in (1) $\alpha - (\frac{3}{4} - \alpha) - \alpha^2 = 0 \quad | \cdot(-1)$

$$\Leftrightarrow \alpha^2 - 2\alpha + 1 - 1 + \frac{3}{4} = 0$$

$$\Leftrightarrow (\alpha-1)^2 = \frac{1}{4} \quad | \sqrt{\ }$$

$$\Leftrightarrow (\alpha - 1) = \pm \frac{1}{2}$$

$$\Leftrightarrow \alpha_1 = \frac{3}{2} \quad \vee \quad \alpha_2 = \frac{1}{2}$$

mit (3) $T = 2 + \frac{3}{4} - \alpha = \frac{11}{4} - \alpha$

folgt $T_1 = \frac{11}{4} - \frac{6}{4} = \frac{5}{4}$, $T_2 = \frac{11}{4} - \frac{2}{4} = \frac{9}{4}$

Stationäre Punkte prüfen:

$W_{\alpha\alpha} = 1 - 2\alpha$, $W_{\alpha T} = W_{T\alpha} = -1$

$W_{TT} = -1$

$$\Rightarrow \Delta = (1-2\alpha)(-1) - (-1)^2 = 2\alpha - 2$$

	α	T	Δ	$W_{\alpha\alpha}$
Lsg 1	$\frac{3}{2}$	$\frac{5}{4}$	$3-2=1$	$-2 \Rightarrow$ Max

Lsg 1	$\frac{3}{2}$	$\frac{5}{4}$	$3-2=1$	$-2 \Rightarrow$ Max
Lsg 2	$\frac{1}{2}$	$\frac{9}{4}$	$1-2=-1$	kein Extremum

Anja erreicht max. W für d = 1.5 Lernstunden/Tag und T = $\frac{5}{4}$ Liter Wunder-trank.

$$W\left(\frac{3}{2}, \frac{5}{4}\right) = \frac{19}{32} + \frac{1}{2}\cdot\frac{9}{4} + \frac{3}{4}\left(-\frac{3}{4}\right) - \frac{3}{2}\left(-\frac{3}{4}\right) - \frac{1}{3}\cdot\frac{3^3}{8} - \frac{1}{2}\left(-\frac{3}{4}\right)^2$$

$$= \frac{19}{32} + \frac{9\cdot 4}{32} - \frac{9\cdot 2}{32} + \frac{9\cdot 4}{32} - \frac{9\cdot 4}{32} - \frac{9}{32}$$

$$= \frac{19 - 18 + 27}{32} = \frac{28}{32} = \frac{7}{8}$$