Aufgabe 8

a) $z_1 = -1 \in \mathbb{C} \qquad z_1 = -1 + 0 \cdot i$

$z = z_1^{1/5} = \sqrt[5]{-1}$

$|z_1| = \sqrt{(-1)^2 + 0^2} = 1 \qquad \sqrt[5]{1} = 1$

$\varphi = 180^\circ$ bzw. π

$z_{11} = 1 \cdot e^{i\,36^\circ}$

$z_{12} = 1 \cdot e^{i(36^\circ + 72^\circ)} = e^{i\,108^\circ}$

$z_{13} = 1 \cdot e^{i(36^\circ + 2 \cdot 72^\circ)} = e^{i\,180^\circ}$

$z_{14} = 1 \cdot e^{i(36^\circ + 3 \cdot 72^\circ)} = e^{i\,252^\circ}$

$z_{15} = 1 \cdot e^{i(36^\circ + 4 \cdot 72^\circ)} = e^{i\,324^\circ}$

$z_{11} = \cos 36^\circ + i \sin 36^\circ = 0.809 + 0.587 i$

$z_{12} = \cos 108^\circ + i \sin 108^\circ = -0{,}309 + 0.951 i$

$z_{13} = \cos 180^\circ + i \sin 180^\circ = -1 + 0 \cdot i = -1$

$z_{14} = \cos 252^\circ + i \sin 252^\circ = -0.309 - 0.951 i$

$z_{15} = \cos 324^\circ + i \sin 324^\circ = 0.809 - 0.587 i$

b) $y' = -x \cdot y^2 \qquad y(0) = 2$

$\frac{dy}{dx} = -x \cdot y^2 \; \Big| \cdot dx$ Trennung der Variablen

$dy = -x \cdot y^2 \cdot dx \quad \Big| : y^2$

$$\frac{1}{y^2} dy = -x \cdot dx \quad \Big| \int$$

$$\int y^{-2} dy = -\int x \, dx$$

$$-y^{-1} = -\frac{x^2}{2} + C$$

$$-\frac{1}{y} = -\frac{x^2}{2} + C \quad \Big| \cdot y$$

$$-1 = y\left(-\frac{x^2}{2} + C\right)$$

$$-1 = y\left(\frac{-x^2 + 2C}{2}\right) \quad \Big| \cdot 2$$

$$-2 = y\left(-x^2 + 2C\right) \quad \Big| : \left(-x^2 + 2C\right)$$

$$y = \frac{-2}{-x^2 + 2C} = \frac{2}{x^2 - 2C}$$

Anfangswert einsetzen: $y(0) = 2$

$$2 = \frac{2}{0 - 2C} = \frac{2}{-2C} \quad \Big| \cdot -2C$$

$$-4C = 2 \Rightarrow C = -\frac{1}{2}$$

Spezielle Lösung: $y = \dfrac{2}{x^2 + 1}$