Lösungen Klausur 24.9.2013 Aufgaben Schmitter

Dienstag, 10. September 2013
18:05

$f(x) = 4\sqrt{1+x} = 4(1+x)^{\frac{1}{2}}$ $f(0) = 4\sqrt{1} = 4$

$f'(x) = \frac{4}{2}(1+x)^{-\frac{1}{2}} = 2(1+x)^{-\frac{1}{2}}$ $f'(0) = 2$

$f''(x) = -\frac{2}{2}(1+x)^{-\frac{3}{2}} = -(1+x)^{-\frac{3}{2}}$ $f''(0) = -1$

$f'''(x) = -\frac{3}{2}(-1)(1+x)^{-\frac{5}{2}} = \frac{3}{2}(1+x)^{-\frac{5}{2}}$ $f'''(0) = \frac{3}{2}$

$f^{(4)}(x) = -\frac{3}{2}\cdot\frac{5}{2}(1+x)^{-\frac{7}{2}}$ $f^{(4)}(0) = -\frac{15}{4}$

Taylorpolynom 3. Grades:

$$T_3(x) = 4 + \frac{2}{1!}x + \frac{(-1)}{2!}x^2 + \frac{3}{2\cdot 3!}\cdot x^3$$

$$= 4 + 2x - \frac{1}{2}x^2 + \frac{1}{4}x^3$$

$$T_3\left(\frac{1}{16}\right) = 4 + 2\cdot\frac{1}{16} - \frac{1}{2}\cdot\frac{1}{16^2} + \frac{1}{4}\cdot\frac{1}{16^3} = 4,1231079101$$

Man berechnet näherungsweise $f\left(\frac{1}{16}\right) = 4\sqrt{1+\frac{1}{16}} = \sqrt{17}$

Restglied: $|R_3(x)| = |f(x) - P_3(x)| \leq \frac{C}{(3+1)!}x^4$

C obere Schranke von $f^{(4)}$ auf $[0, \frac{1}{16}]$

$f^{(4)}$ monoton wachsend, daher $\left|-\frac{15}{4(1+\frac{1}{16})^{\frac{7}{2}}}\right| < 3$

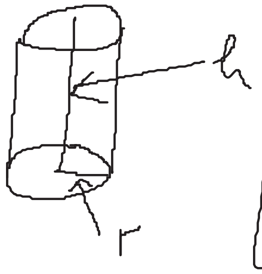
$$|R_3(x)| \leq \frac{3}{4!}\cdot\frac{1}{16^4} = 0.000001907349$$

$\Rightarrow$ Genauigkeit auf 5 Dez.stellen exakter Wert: $4.12310|5625618$

Näherungswert: $4.\underbrace{12310}_{5\text{ Stellen}}|79\ldots$

(s.o.)

c)



$V = \pi \cdot r^2 \cdot h = 5000\,cm^3$

$O_b: \pi \cdot r^2 + 2\pi r h$

$\rightarrow h = \dfrac{5000}{\pi \cdot r^2}$

$$O_b(r): \pi r^2 + 2\pi r \cdot \frac{5000}{\pi \cdot r^2}$$

$$= \pi r^2 + \frac{10000}{r}$$

$$O_b'(r) = 2\pi r - \frac{10000}{r^2}$$

$$O_b''(r) = 2\pi + \frac{20000}{r^3}$$

$$O_b'(r) = 0 \Leftrightarrow 2\pi r = \frac{10000}{r^2} \quad | \cdot r^2$$

$$2\pi r^3 = 10000 \quad | : 2\pi$$

$$r^3 = \frac{10000}{2\pi} \quad | \sqrt[3]{}$$

$$r = \sqrt[3]{\frac{5000}{\pi}} \approx 11.67\,cm$$

$O_b''(r) > 0 \Rightarrow$ Min $\quad h = \dfrac{5000}{\pi \cdot r^2} \approx 11.67\,cm$

Aufgabe 4