a) Rang: Anzahl maximal lin. unabh. Zeilen (bzw. Spalten)

b)
$$\begin{pmatrix} 2 & 6 & 3 & 2 \\ 1 & 1 & 2 & 4 \\ 1 & 2 & 4 & 7 \\ 2 & 3 & 3 & 5 \end{pmatrix} \begin{matrix} \\ -1/2 \times Z1 \\ -1/2 \times Z1 \\ -1 \times Z1 \end{matrix}$$

$\downarrow$

$$\begin{pmatrix} 2 & 6 & 3 & 2 \\ 0 & -2 & 1/2 & 3 \\ 0 & -1 & 5/2 & 6 \\ 0 & -3 & 0 & 3 \end{pmatrix} \begin{matrix} :2 \\ :(-2) \\ \cdot(-1) & -Z2 \\ :(-3) & -Z2 \end{matrix}$$

$\downarrow$

$$\begin{pmatrix} 1 & 3 & \frac{3}{2} & 1 \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{2} \\ 0 & 0 & -\frac{9}{4} & -\frac{9}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix} \begin{matrix} \\ \\ \cdot -\frac{4}{9} \\ \cdot 4 \end{matrix}$$

$\downarrow$

$$\begin{pmatrix} 1 & 3 & \frac{3}{2} & 1 \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{2} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{pmatrix} -3Z$$

$\downarrow$

$$\begin{pmatrix} 1 & 3 & \frac{3}{2} & 1 \\ 0 & 1 & -1/4 & -\frac{3}{2} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$rg = 3$

$$\left(\begin{array}{cccc} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad y = 0$$

c) Lösungsvektor am Ende: $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{pmatrix}$

Aufgabe 5

$$df = f_x\,dx + f_y\,dy$$

$$f_x = \frac{x}{\sqrt{x^2+y^2}} \qquad f_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$dx = -0.02$$

$$dy = +0.01$$

$$df = f_x(3,4)\cdot(-0.02) + f_y(3,4)(0.01)$$

$$= \frac{3}{5}(-0.02) + \frac{4}{5}(0.01)$$

$$= -0.004$$

$$f(3,4) = \sqrt{3^2+4^2} = \sqrt{25} = 5$$

$$f(2.98, 4.01) = \sqrt{2.98^2+4.01^2} = 4.99604$$

b) $V_x = \pi \int_0^{\pi} \sin x \cdot \sin x\,dx$