Aufg. 1

a) $\sqrt{5-x} = x-3 \quad | ()^2$

$\Rightarrow (5-x) = x^2-6x+9$

$\Leftrightarrow \quad 0 = x^2-5x+4$

$\Leftrightarrow \quad 0 = (x-1)(x-4)$

$\Rightarrow$ Kandidaten sind $x_1 = 1$ u. $x_2 = 4$

Probe: $\sqrt{5-1} = 1-3$ ✗

$\sqrt{5-4} = 4-3$ ✓

$\Rightarrow \underline{\underline{L = \{4\}}}$

b) $(3\cdot7 + 81^{1285} - 27\cdot16) \bmod 8$

$= (5 + 1 - 3\cdot0) \bmod 8 = \underline{\underline{6}}$

$((21\cdot16)^{250} + 3 - 3\cdot15) \bmod 5$

$= (1\cdot1 + 3 - 3\cdot0) \bmod 5 = \underline{\underline{4}}$

c) $9^{500} = (2^{ld 9})^{500} = 2^{ld 9\cdot500}$, also

Stellen binär: $\underbrace{\left\lceil \frac{\ln 3}{\ln 2}\cdot500 \right\rceil}_{\lceil ld 9\cdot500\rceil} = \lceil 1584.96 \rceil = \underline{\underline{1585}}$

($\lceil x \rceil$ = Gaußklammer = nächster ganzz. Wert über x)

Aufg 2

a) $\frac{10\cdot 0 - 5\cdot 0 + 3}{\cos(0)} \lim_{x\to 0}\left(\frac{\sqrt{x+4}-2}{2x}\right) \overset{\substack{0/0\text{-Sit, L'Hospital}\\ \downarrow}}{=}$

$= \frac{3}{1} \cdot \lim_{x\to 0}\left(\frac{\frac{1}{2\sqrt{x+4}} - 0}{2}\right) = 3 \cdot \frac{\left(\frac{1}{4}\right)}{2} = \underline{\underline{\frac{3}{8}}}$

b) $\lim_{x\to\infty} \frac{\ln(5x+3)}{e^x} \underset{\substack{\uparrow\\ \frac{\infty}{\infty}\text{-Sit}}}{=} \lim_{x\to\infty} \frac{\left(\frac{1\cdot 5}{5x+3}\right)}{e^x}$

$= \lim_{x\to\infty} \frac{5}{(5x+3)e^x} = \frac{5}{\infty\cdot\infty} = \underline{\underline{0}}$

c) Zu fordern: $25 - x^2 \geq 0 \iff 25 \geq x^2$

$\iff |5| \geq |x| \iff x \in [-5, 5]$

und $4 - \sqrt{25-x^2} \geq 0 \iff 4 \geq \sqrt{25-x^2} \quad |()^2$

$\Rightarrow 16 \geq 25 - x^2 \iff x^2 \geq 9 \iff |x| \geq 3$

$\iff x \in]-\infty, -3] \cup [3, \infty[$

Insgesamt $D = [-5, -3] \cup [3, 5]$

Aufg. 7

a) Median $m = \frac{89+90}{2} = 89.5$

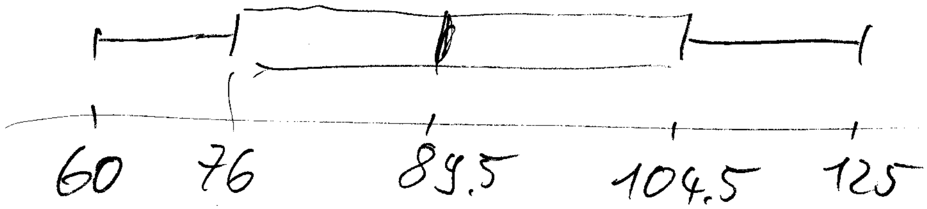
0.25-Quartil $= \frac{75+77}{2} = 76$

0.75- " $= \frac{102+107}{2} = 104.5$

$IQR = 104.5 - 76 = 28.5$

unterer Whisker $76 - 1.5\,IQR = 33.25$
→ Datenpunkt 60

oberer Whisker $104.5 + 1.5\,IQR = 147.25$
→ Datenpunkt 125



c) $p = 0.8$, $N = 1000$, $\mu = np = 800$

$\sigma = \sqrt{p(1-p)N} = 12.65 = 4\sqrt{10}$

Vorrauss. Moivre-Lapl. erfüllt: $np = 800 > 5$
$n(1-p) = 200 > 5$

$P(X \geq 790) = 1 - P(X < 790)$

$= 1 - \Phi\left(\frac{790 - 800 - 0.5}{\sigma}\right) =$

$= 1 - \Phi\left(-\frac{10.5}{\sigma}\right) = 1 - \left(1 - \Phi\left(\frac{10.5}{\sigma}\right)\right) = \Phi\left(\frac{10.5}{\sigma}\right)$

$= \Phi(0.83) \approx 0.797 = \underline{\underline{79.7\%}}$

Aufg. 76)

(i) 8 b-Positionen aus Positionsmenge: ZoZ, Reihenfolge egal

$$\binom{10}{8} = \frac{10 \cdot 9}{1 \cdot 2} = 45$$

2 Positionen mit bel. ≠b besetzen

$$4 \cdot 4 = 16$$

Insgesamt: $45 \cdot 16 = \underline{\underline{720}}$

(ii) 9 b's: $\binom{10}{9} \cdot 4 = 10 \cdot 4 = 40$

10 b's: $\binom{10}{10} = 1$

Insgesamt: $\underline{\underline{761}}$

Aufg. 8

a) $z^8 = 256\,i = 256\, e^{i\frac{\pi}{2}} \quad | ()^{\frac{1}{8}}$

$z = 256^{1/8}\, e^{i(\frac{\pi}{2} + 2k\pi)\cdot\frac{1}{8}}$

$= \underline{\underline{2\, e^{i(\frac{\pi}{16} + k\frac{\pi}{4})}}} \quad , \quad k = 0, \dots, 7$

b) 2. Ordnung, linear, inhomogen, mit konst. Koeff., explizit

c) Ansatz homogene DGL: $y_h = C e^{\lambda x}$

$\Rightarrow y_h'' = \lambda^2 y_h$. Einsetzen in homogene DGL

$\lambda^2 \cancel{4} C e^{\lambda x} + 4 C e^{\lambda x} = 0 \iff \lambda^2 = -4$

$\iff \lambda = \pm 2i$, zwei Lsg.

$\Rightarrow \underline{y_h(x) = C_1 e^{2ix} + C_2 e^{-2ix}}$

d) Partikuläre inhom. Lsg.: $y_p = D = \text{const}$

$y_p'' = 0$

$\Rightarrow 4D = 8 \iff D = 2$

$\Rightarrow \underline{\underline{y(x) = C_1 e^{2ix} + C_2 e^{-2ix} + 2}}$