

## Aufgabe 3 a

$$f(x) = \sqrt{1+x} = (1+x)^{\frac{1}{2}}$$

$$f(0) = \sqrt{1} = 1$$

$$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}$$

$$f'(0) = \frac{1}{2} \cdot \frac{1}{\sqrt{1}} = \frac{1}{2}$$

$$f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}}$$

$$f''(0) = -\frac{1}{4} \cdot \frac{1}{\sqrt{1^3}} = -\frac{1}{4}$$

$$f'''(x) = \frac{3}{8}(1+x)^{-\frac{5}{2}}$$

$$f'''(0) = \frac{3}{8} \cdot \frac{1}{\sqrt{1^5}} = \frac{3}{8}$$

$$f^{(4)}(x) = -\frac{15}{16}(1+x)^{-\frac{7}{2}}$$

$$f^{(4)}(0) = -\frac{15}{16} \cdot \frac{1}{\sqrt{1^7}} = -\frac{15}{16}$$

$$\begin{aligned} T_4(x) &= \frac{f(0)}{0!} x^0 + \frac{f'(0)}{1!} x^1 + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4 \\ &= 1 + \frac{1}{2}x - \frac{1}{4 \cdot 2}x^2 + \frac{3}{8 \cdot 6}x^3 - \frac{15}{16 \cdot 24}x^4 \end{aligned}$$

$$T_4(0.2) \approx \sqrt{1+0.2} = \sqrt{1.2}$$

$$= 1 + \frac{1}{2} \cdot 0.2 - \frac{1}{8} 0.2^2 + \frac{3}{48} 0.2^3 - \frac{15}{384} 0.2^4$$

$$= 1 + 0.1 - 0.005 + 0.0005 - 0.0000625$$

$$= \underline{1.0954375}$$

$$\sqrt{1.2} = 1.095445115$$

Im Taschenrechner ist die Wurzelberechnung durch ein Taylorpolynom höheren Grades "abgemacht"