Beispiel:

$$z = \underbrace{-2}_{x} + \underbrace{2\sqrt{3}}_{y}\, i$$ Was ist r und φ?

Lsg: $r = \sqrt{x^2+y^2} = \sqrt{(-2)^2 + 2^2 3} = \sqrt{4+12} = \underline{\underline{4}}$

Welcher Quadrant? 2. ~~Qu~~ Quadrant (x<0)

$$\varphi = \arctan\left(\frac{2\sqrt{3}}{-2}\right) + \pi = \frac{2}{3}\pi \mathrel{\hat{=}} 120^\circ$$

Übung

$$z = 1.5 - 1.5\sqrt{3}\, i$$ Was ist r und φ?

Lsg: $r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\sqrt{3}\right)^2} = \sqrt{\frac{9}{4} + \frac{9}{4}\cdot 3} = \sqrt{\frac{9+27}{4}}$

$= \sqrt{9} = \underline{\underline{3}}$

z liegt im 4. Quadrant ($x > 0$, $y < 0$)

$$\Rightarrow \quad \varphi = \arctan\left(\frac{-\frac{3}{2}\sqrt{3}}{\frac{3}{2}}\right) = \arctan\left(-\sqrt{3}\right)$$

$$= -\frac{\pi}{3} \mathrel{\hat{=}} -\frac{\pi}{3} + 2\pi = \frac{-\pi + 6\pi}{3} = \underline{\underline{\frac{5\pi}{3}}}$$

(ii)

Welche Lösungen hat $z = (1+i)^{\frac{3}{4}}$

Hinweis: 1. Schritt $(1+i)$ in Exp.darstellung

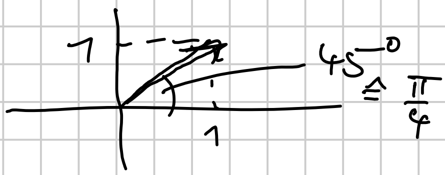
$y = 1+i$ $\qquad r = \sqrt{Re(y)^2 + Im^2(y)}$

$$= \sqrt{1^2 + 1^2} = \underline{\underline{\sqrt{2}}}$$

y liegt im 1. Quadranten

$$\varphi = \arctan\left(\frac{Im(y)}{Re(y)}\right) = \arctan 1$$

$$\Rightarrow \varphi = \underline{\underline{\frac{\pi}{4}}}$$



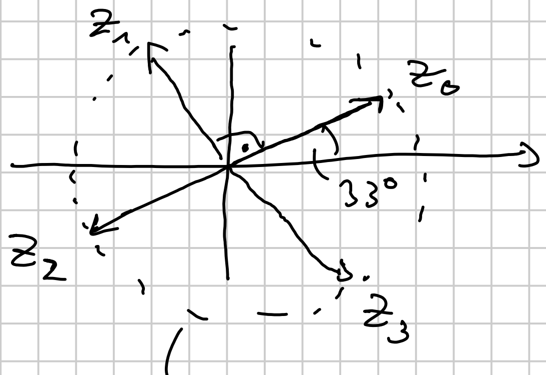
$$z = (1+i)^{\frac{3}{4}} = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^{\frac{3}{4}}$$

$$= \left(\left(2^{\frac{1}{2}}\right)^3 e^{i\left(\frac{3\pi}{4} + 2k\pi\right)}\right)^{\frac{1}{4}}$$

$$= \underline{\underline{2^{\frac{3}{8}}\, e^{i\frac{3\pi}{16} + k\frac{\pi}{2}}}}$$

$k = 0, 1, 2, 3$

$\frac{3\pi}{16} \hat{=}$ 33.75°



Radius $2^{\frac{3}{8}}$