Wdh

Mittwoch, 5. Mai 2021 10:56

Erwartungswert

"Wert den ich im Durchschnitt erwarte (viele Wdh)"

Bsp. Würfel: $\{1, 2, 3, 4, 5, 6\}$ $\quad p_m = \frac{1}{6}$ $\quad \leftarrow x_m$

$E(X) = 3.5$

X diskret: $E(X) = \sum_m x_m p_m$

X stetig: $E(X) = \int_{-\infty}^{\infty} t \, w(t) \, dt$

Linearität $\quad Z = aX + b$

$E(Z) = a E(X) + b$

$Var(Z) = a^2 Var(X)$

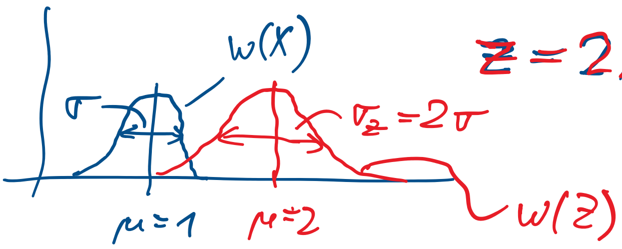
$\sigma(Z) = a \, \sigma(X)$

$Var(X) = E((X-\mu)^2)$

$\sigma(X) = \sqrt{Var(X)}$

$\sigma^2(X) = Var(X)$

$Z = 2X$



Wann Binomial-Vert: genau zwei Ausgänge

Bi-Nomial = zwei Namen

Unterschied Binomial $\longleftrightarrow$ Hypergeom

ZmZ $\longleftrightarrow$ ZoZ

Regeln Gauss

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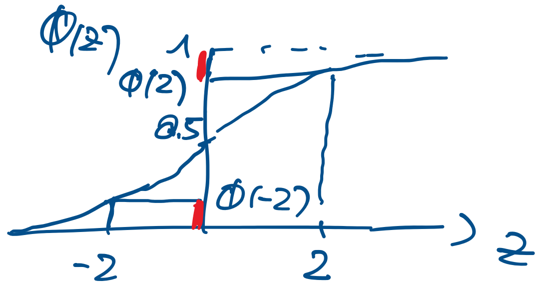
1) $\Phi(-z) = 1 - \Phi(z)$

$\Phi(-2) = 1 - \Phi(2)$



2) Ist X $N(\mu, \sigma^2)$ verteilt, dann ist

$\underline{Z} = \frac{X-\mu}{\sigma}$ $\underline{N(0,1^2) \text{ verteilt}}$ → Tabelle Φ

3) $P(X \leq b)$ (z.B. $b=2$)

$$= P\left(\frac{X-\mu}{\sigma} \leq \frac{b-\mu}{\sigma}\right) \underset{Nr.2}{=} P\Big(Z \leq \underbrace{\frac{b-\mu}{\sigma}}_{z}\Big) = \Phi\left(\frac{b-\mu}{\sigma}\right)$$

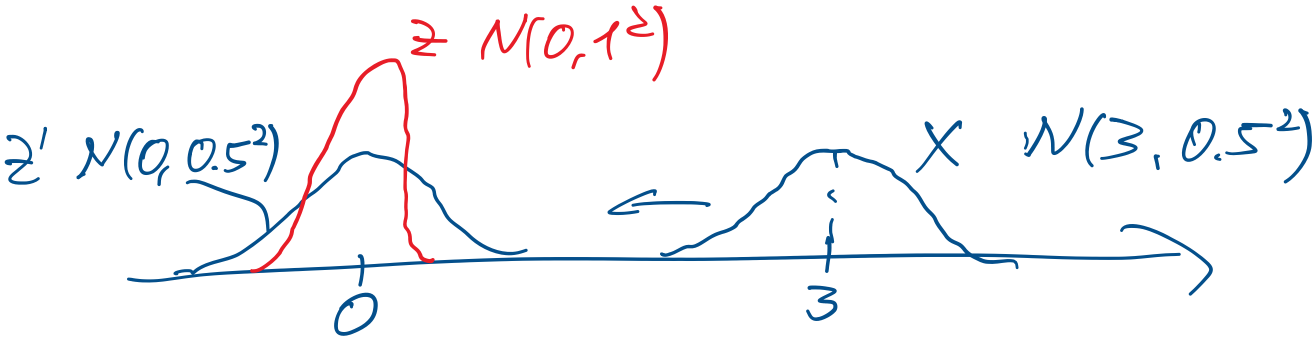
standard normalvert.

X $N(3, 0.5^2)$-verteilt $\Rightarrow \Phi\left(\frac{b-\mu}{\sigma}\right)$

Konkret: $\mu = 3$, $\sigma = 0.5$, $b = 2$

$$P(X \leq 2) = P\left(Z \leq \frac{2-3}{0.5}\right) = P(Z \leq -2)$$
$$= \Phi(-2)$$
$$\underset{Nr.1}{=} 1 - \Phi(2) \rightarrow \text{Tab.}$$

Nochmal zu 2



Bsp: $\Phi(-1.3) = 1 - \underbrace{\Phi(1.3)}_{\text{Tab } 0.9032} = 0.0968$

Regeln Gauß 2

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X $N(\mu,\sigma^2)$-verteilt

4) $P(a \leq X \leq b) = F(b) - F(a)$

$$\overset{2x\ \text{Nr.}3}{=} \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

5) $q = \Phi(z_q) \iff 1-q = \Phi(-z_q)$

Bew. $1-q = \Phi(-z_q)$

$\overset{\text{Nr.}1}{\iff} 1-q = 1-\Phi(z_q)$

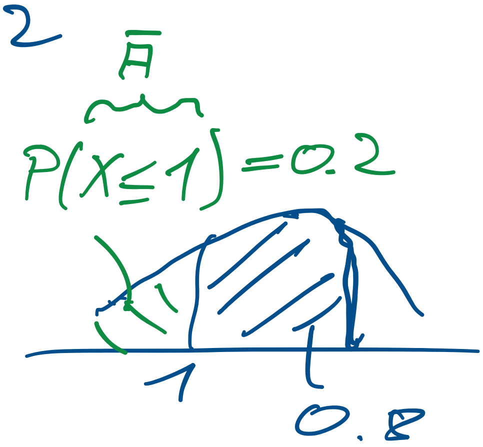
$\iff q = \Phi(z_q)$

Rezepte: Wenn $P(X \geq 1) = q' = 0.8$ geg.

$\rightarrow P(X \leq 1) = 1 - q' = 0.2 = q$

Suche z_q für $q = 0.2$

Wenn $P(\underbrace{X \geq 1}_{A}) = 0.8$



Beispiel Gauss

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Bsp 0) Geg sei normalverteiltes X mit $\mu = 4$ und $\sigma = 2$. $X \sim N(4, 2^2)$

Wie groß ist a) $P(X \leq 6)$ b) $P(X \geq 2)$

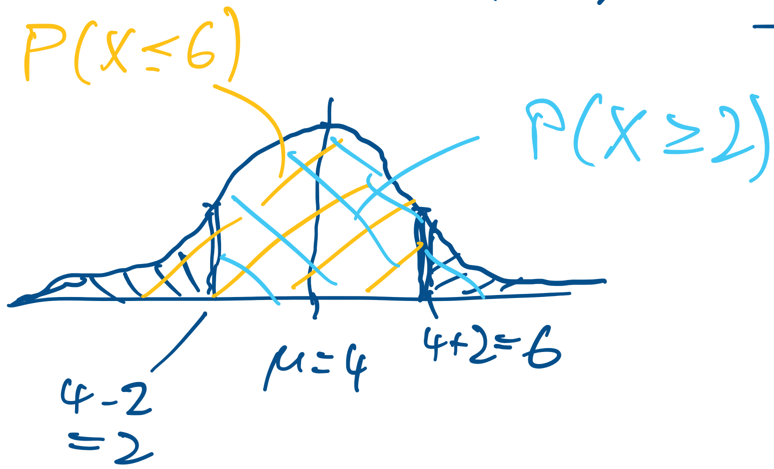
c) $P(3.8 < X \leq 7)$?

Lsg: a) $P(X \leq 6) = F(6) \overset{Nr.3}{=} \Phi\left(\frac{6-4}{2}\right) = \Phi(1) \overset{Tab}{=} 84.1\%$

b) $P(X \geq 2) = 1 - P(X \leq 2) = 1 - F(2) \overset{Nr.3}{=}$

$1 - \Phi\left(\frac{2-4}{2}\right) = 1 - \Phi(-1) \overset{Nr.1}{=} 1 - (1 - \Phi(+1))$

$= \Phi(1) = \underline{\underline{84.1\%}}$



c) $P(3.8 \leq X \leq 7) \overset{Nr.4}{=} \Phi\left(\frac{7-4}{2}\right) - \Phi\left(\frac{3.8-4}{2}\right)$

$= \Phi(1.5) - \Phi(-0.1)$

$\overset{Nr.1}{=} \Phi(1.5) - (1 - \Phi(0.1)) \overset{Tab}{=} 47.3\%$

$= 0.9332 - (1 - 0.5398)$