

Beweis zu S 10-8

$$1. \lim_{t \rightarrow -\infty} F(t) = 0, \text{ denn } \lim_{t \rightarrow -\infty} F(t) = P(X \leq -\infty) = P(\{\}) = 0$$

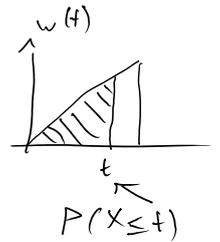
$$\lim_{t \rightarrow +\infty} F(t) = 1, \text{ denn } \lim_{t \rightarrow +\infty} F(t) = P(X < +\infty) = P(\Omega) = 1$$

$$2. P(a < X \leq b) = F(b) - F(a) \quad \text{denn}$$

$$P(X \leq a) + P(a < X \leq b) = P((X \leq a) \vee (a < X \leq b)) = P(X \leq b) \quad \Rightarrow$$

$\begin{matrix} \rightarrow & \leftarrow \\ \text{un vereinbar} \end{matrix}$

$$\begin{aligned} P(a < X \leq b) &= P(X \leq b) - P(X \leq a) \\ &= F(b) - F(a) \end{aligned}$$



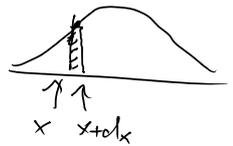
Zur Formel Erwartungswert stetig

$$\text{diskret: } E(X) = \sum_m x_m p_m$$

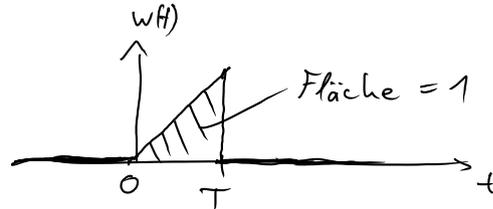
$$p_m = P(X = x_m)$$

$$\text{stetig: } E(X) = \int_{-\infty}^{\infty} x w(x) dx$$

$$w(x) dx = P(x < X < x + dx)$$



$$w(t) = \begin{cases} \alpha t & 0 < t \leq T \\ 0 & \text{sonst} \end{cases}$$



Parameter T ist wie die 5 im vorigen Beispiel (wie Parameter in einem Programm)

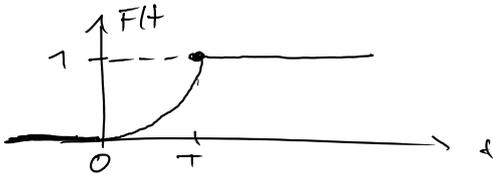
Bestimme α über Bedingung "Fläche = 1"

$$1 = \int_{-\infty}^{\infty} w(t') dt' = \int_0^T w(t') dt' = \int_0^T \alpha t' dt' = \frac{1}{2} \alpha t'^2 \Big|_0^T = \frac{1}{2} \alpha T^2 \Rightarrow$$

$\alpha = \frac{2}{T^2}$

Stammfkt

$$F(t) = \int_{-\infty}^t w(t') dt' = \frac{1}{2} \alpha t'^2 \Big|_0^t = \frac{1}{2} \alpha t^2 = \frac{1}{2} \frac{2}{T^2} t^2 = \frac{t^2}{T^2} \quad \text{für } 0 < t \leq T$$



4 Erwartungswert $\mu = E(x) = \int_0^T t \cdot \alpha t dt = \int_0^T t \frac{2}{T^2} t dt = \frac{2}{T^2} \int_0^T t^2 dt$

$$= \frac{2}{T^2} \frac{1}{3} t^3 \Big|_0^T = \frac{2}{3} T$$

